Structural Engineering Documents

5

3rd reviewed and extended Edition

Jörg Schneider
Ton Vrouwenvelder

Introduction to Safety and Reliability of Structures

Including free access to
Variables **P**rocessor Software **FreeVaP**

International Association for Bridge and Structural Engineering (IABSE)

Jörg SCHNEIDER

Born in 1934 in Cologne, Germany, Jörg Schneider received his civil engineering degree from the Swiss Federal Institute of Technology Zurich (ETHZ) in 1958. He was an assistant at ETH from 1959 to 1963 and then joined the firm of Stahlton AG, where he was involved in the design and development of prestressed and precast concrete structures.

Since 1967 he has been professor for Structural Engineering at ETHZ. His research interests include safety and reliability of structures, with special emphasis on human error. He retired from ETHZ in 1999.

He joined the IABSE in 1968 and was active in many of its committees and one of its vice-presidents from 1993 to 2001. He has been a member of the Joint Committee on Structural Safety from 1979 to 2002 and its president from 1990 to 1994. In 1998 he received the Dr.h.c. by the University of Natural Resources and Life Sciences, Vienna. In 2002 he was elected honorary member of IABSE.

In 1999 Jörg Schneider founded, together with a few friends, the consulting office Risk&Safety AG, Aarau, Switzerland.

Ton VROUWENVELDER

Ton Vrouwenvelder was born in 1947 in The Hague, the Netherlands. He received his diploma in civil engineering at the Technical University of Delft in 1970 cum laude. As a junior researcher he worked at the Applied Mechanics Division till 1977 and then moved to the Concrete Department of TNO-Bouw.

In 1987 he became part time professor at Delft University, gave courses in structural mechanics and started up a course on probabilistic methods in civil engineering together with professor Han Vrijling.

He has been involved in national and international research and consultancy projects as well as in the development of standards codes for design and assessment of buildings and civil engineering structures (Dutch standards as well as ISO standards and Eurocodes). In particular he has been involved in the development or Eurocode 1990 and ISO 2394. He is a member of the Joint Committee on Structural Safety since 1982 and served as its president in the period from 1999 till 2005. In 2011 he received the C. Allin Cornell CERRA Award.

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Preface

The book you hold in your hands is based on IABSE's Structural Engineering Document SED 5, published in 1997 upon request of Working Commission I of the IABSE and supported by the Joint Committee on Structural Safety (JCSS) in order to advance the use of reliability methods in structural engineering. The book offers a short, informative, and more educational type of text and figures on safety and reliability analysis in structural engineering. It is intended both for students and practising engineers and aims to keep things understandable and to explain concepts and procedures by simple examples rather than by digging deep into the theory. Thus, almost no proofs are given. It is hoped that this book serves its purpose in furthering a topic which is gaining more and more attention and finding increasing application in practice.

The text and figures are based on parts of the lecture course "Sicherheit und Zuverlässigkeit im Bauwesen" given by *J. Schneider* in the 90th to 3rd year students in the Civil Engineering Department of the Swiss Federal Institute of Technology, Zurich (ETHZ). This course was very much influenced by a short course given in Zurich by *A. Nowak* in *1987*.

Translations from the German were done by *E.G. Prater* of ETHZ, and *Hillary Hart* of the University of Texas at Austin, U.S.A. A number of members of IABSE's Working Commission I and of the JCSS carefully read the text pointing out mistakes and suggested shortening and amending here and there. Among those whose help is gratefully acknowledged are *T. Vrouwenvelder* and *R. Rackwitz.*

SED 5 was well received in 1997. A 2nd edition was printed in 2006, and, grace to the permission of *M. Petschacher*, was supplemented with a free educational type of Variables Processor software, *FreeVaP*, in order to help in understanding the subjects treated.

The second edition was sold out in 2016. In view of the facts that the book turned out to be very attractive for starters who do not want to be overwhelmed by too heavy mathematics, and that the book sells well, IABSE decided to print a third edition.

However, during the last 20 years quite some progress was observed and the first author found himself not really up-to-date anymore to cope with all of these developments. He was very happy to find in the second author a good friend of former times and a person fully knowledgeable in all necessary fields to bridge all gaps. In good co-operation of the undersigned a number of new chapters were introduced and additions and corrections here and there were made resulting in some 40% increase of volume. And, again, access to the software mentioned above was ensured.

The feedback by the reviewers of the manuscript was well received and is gratefully acknowledged. A very special thank-you of the authors goes to Mikael Breastrup for his very careful look at the contents of this book and for bringing quite a number of larger and smaller blunders, errors and mistakes to the surface. This greatly enhances the value of this book for an inexperienced reader as she or he is not unnecessarily puzzled by errors in the book itself, once there is a beginning of real understanding.

We wish this book a good start into the next decade.

Zurich, Spring 2017

Jörg Schneider, Zurich
and
Ton Vrouwenvelder, Delft

Content overview

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1. Dealing with Hazards

1.1 Main concepts

In this chapter the most important terms in structural safety and structural reliability are discussed and defined. In so doing, a rather broad approach to the safety of structures and related topics is taken.

1.11 Safety

Society expects that the occupants and users of buildings and structures and persons in their vicinity or area of influence are safe. People expect that the failure of buildings and structures is extremely rare and they consciously rely on the professional care and expertise of those involved in the planning, design, analysis, detailing, construction, and maintenance of structures.

The definition of the term *safety* must consider these facts and expectations. Thus, in this book, safety is defined as follows:

- The term safety is primarily related to the safety of people affected by structural failures.
- Adequate safety with respect to a hazard is ensured provided that the hazard is kept under control by appropriate measures or the respective risk is limited to an acceptable value.
- Absolute safety is not achievable.

Safety in the above sense – as opposed to the term *risk* – is obviously a *qualitative* term. Safety is achieved if the risk of damage to persons is reduced to comparatively small and thus acceptable values. The definition includes the safety of these groups:

- the workers at the site,
- the users of a structure or a facility,
- other persons in the vicinity of a structure, a facility or the environment at large.

It is important to note that in the above definition, it is not the structure the facility or the environment as such that is designated safe, but rather the people in the respective area of influence.

Safety problems attached to items, systems, facilities or events generally can be identified by simply asking the question: "Are persons endangered if this item, system, or facility fails or this specific event occurs"? If the answer is „yes“, then utmost care is requested.

Typical safety problems, therefore, result from the failure of a residential or commercial building, or a bridge, but they can also result from events such as train collisions, the sinking of ships. Also the collapse of a fuel tank endangering the human environment, etc., might fall in this category. Considered in this way, the collapse of an empty tower on a lonely hill during prolonged snowstorms is not a safety problem, being sure that, in such circumstances, no one is in the area.

1.12 Reliability

Reliability is defined as the probability that an item or facility will perform its intended function for a specified period of time, under defined conditions. The item under consideration could be a structure, a lamp, or a coffee maker. The probability of failure is denoted by p_f. In view of this definition the reliability r is defined as the complement of the probability of failure p_f:

$$r = 1 - p_f \tag{1.1}$$

In contrast to safety, reliability is measurable, i.e., quantifiable. Lack of reliability implies that a condition, with a certain probability, will not be fulfilled: e.g., a structure does not collapse, or the existing deflections do not exceed values deemed to be permissible, or reinforcing bars do not rust prematurely, etc.

1.13 Underlying concepts

a) Probability and frequency

Events, e.g., the event A = (missing a train), occur with a certain probability, a term which basically can be defined in three different ways:

- *Classical (Laplace):* Probability is the number of cases in which an event occurs, divided by the total number of possible cases.
- *Frequency (von Mises):* Probability is the limiting case of the relative frequency with which an event occurs, considering many independent recurrences under the same conditions.
- *Subjective (Bayes):* Probability is the degree of belief or confidence of an individual in the statement that a possible event occurs.

Probabilities p, and conditional probabilities as well, are dimensionless and exhibit a value between zero and one. For probabilities of events which are related to a certain time interval, one speaks more correctly of frequencies, thus e.g., f(A) = 5/year. Since such frequencies of occurrence are often very small, e.g., f(A) = 0.02/year, they are sometimes confused with probabilities.

As there are not many structures of exactly the same kind and loading, it is obvious from the above definitions that for structural reliability the subjective perception of probability is the only feasible one. This statement should, however, not be understood to imply that structural safety or structural reliability are subjective in a sense that excludes rational reasoning and data. The importance of reasoning will certainly be seen from the contents of this book.

b) Risk

The term risk is a measure for the severity of a hazard. The two constituents of risk are the probability p_f or the frequency f_f of a damaging event A and the so-called average or expectation E(D|A) of the damage should this event take place. E(D|A) may be expressed in monetary units or in terms of casualties, e.g., injured or dead people per event (e.g., 3.4 injured per event as a result of 51 injured people in 15 accidents), or by some other damage indicator.

The simplest function relating the two constituents of risk is the product of these quantities:

$$R = p_f \cdot E(D|A) \quad \text{or}$$

$$R = f_f \cdot E(D|A) \tag{1.2}$$

This definition is useful in many practical cases (e.g., in actuarial calculations). Since probabilities are dimensionless quantities, risk has the dimension of the damage quantity itself. If risk is related to a particular time interval, e.g., to a year, then risk is related to time (e.g., R = 51 injured people per year).

Difficulties arise with very small probabilities or frequencies of occurrence of events with very large expectations of damage. Here the product rule fails ("zero-times-infinity-dilemma") as $0 \cdot \infty$ mathematically may take on any value. But also seen from a practical point of view extremely low probabilities and/or extremely large consequences of possible events may be a subject of concern. In such cases the consideration of the maximum possible damage is often decisive. Sometimes this may lead to the conclusion that, although the probability of an adverse event and thus the respective risk is very small, an extremely hazardous activity has to be abandoned because the maximum possible damage is judged to be unacceptable.

In many cases risks have to be considered in more detail. Terms like acceptable risks, voluntary and involuntary accepted risks, and (frequently misunderstood) residual risks are clearly to be distinguished from each other, as are individual and collective risks. It should also be observed that the subjective perception of risk often differs considerably from the objective risk (whatever that is). Chapter 5 is going into more detail.

c) How safe is safe enough?

To answer the above basic question posed by *Starr, 1969* and by *Fischhoff et al., 1978* presupposes the answering of a series of preparatory questions.

First of all the question assumes that *all hazards* are *recognised* which lurk in any particular situation. In a further step it is a question of analysing the corresponding risks. *"What could happen in what way and how often?"* is the question which one has to tackle objectively and free from any value judgements. The next and much more difficult question to answer is "What *may be allowed to happen, how often, and where?"*. Answering that question calls for judgement and for decisions regarding responsibilities.

A comparison of the answers to the above two questions is an assessment: "Is the unfavourable event and its frequency of occurrence acceptable? Is the situation sufficiently safe?"

Usually, for the engineer, if the assessment turns out to be negative, further questions have to be considered, i.e., what suitable measures are needed to provide the required safety. He/she is then the one who has to plan these measures, put them into action and supervise proper functioning.

It goes without saying that safety measures cost money. Therefore, safety essentially is a matter not only of risks and consensus about acceptable risks, but also of cost (see section 5.2).

It is clear that in considering the above questions society is confronted with risks in quite different problem areas. Besides risks from the traditional areas of forces of nature, home, work, traffic, etc. there are also risks related to civilisation's growth (*Schneider, Th., 1981, 1991, or 1992*), such as transport of hazardous goods, adverse events in long tunnels, or risks posed by the chemical industry, and perceived risks from nuclear energy or gene technology.

Structural engineering obviously belongs to those problem areas which are so bound up with our daily life that society normally does not realise that there might be problems. Generally, risks from failing structures do not generate discussion.

d) Optimal design

The design of a structure or any other item or facility may follow optimisation strategies such that the overall cost accumulated throughout the life of the structure or item, including the cost of a possible failure, is minimal.

This may formally be written as

$$C = C_P + C_E + C_M + C_R + p_f \cdot E(D) \rightarrow \text{Minimum} \tag{1.3}$$

Herein is

C_P = cost of planning
C_E = cost of execution
C_M = cost of operation and maintenance
C_R = cost of demolition and restoration of original state

while the last term in expression (1.3) represents the possible costs of a failure of the structure during its service life given by the product of failure probability p_f and the expectation of damage E(D). From this expression, theoretically at least, a target probability of failure or, better, a target reliability β_0 (see section 5.3) could be derived.

This target, however, is not so easy to determine, for two reasons. The first is that probabilities of failure of structures are usually very small and therefore numerical values depend very much on assumptions and methods of reliability analysis. In fact, probabilities of failure or reliability indices are notional in a sense that they may well be used for a relative classifying of designs from good to less good. Such values should, however, not be misinterpreted as representing absolute values.

The second reason introduces even more doubts about optimisation expressed by eqn. (1.3) because actual probabilities of failure of structures are essentially governed by *human error*. Of course, theoretically at least, human error could also be taken into account in calculating failure probabilities. Normally, however, this is not done and therefore calculated failure probabilities are – again – to be regarded as notional.

Results of analysis being notional only rather than reflecting truth does not mean, however, that structural reliability concepts and reliability analysis are impractical. As long as the assumptions and methods of reliability analysis are standardised and the results are interpreted in a comparative way, probabilistic concepts and procedures supply very useful information.

1.2 Hazards in structural engineering

Obviously, a closer look is justified at what is going on in the building sector with respect to failures, damage, and possible countermeasures.

1.21 Findings from 800 failure cases

800 cases of damage to structures were analysed, looking for causes and possible countermeasures (*Matousek & Schneider, 1976)*. The most important findings of this study are the following.

Most types of damage already appear in the execution phase and may be traced back to influences which have nothing to do with the structure's later use – on which engineers usually concentrate their attention. This fact should cause engineers to devote more attention to questions, problems, and influences relating to execution.

Some important additional findings are summarised in fig. 1/1. The columns in the table contain the percentages of the number of damage cases (N), of the total damage cost (D), and of all casualties (C), that fall into the corresponding category. A number of lessons may be learned from the table.

Structural systems, temporary structures (scaffolding, etc.), excavations, and site installations cover the majority of the cases, and with almost 90% of the amount of damage they are the major damage factor. What stands out is that more than 80% of all casualties are connected with the failure of these structural components. This should lead us to devote special care to planning, analysis, dimensioning and detailing of structures.

	N	D	C
Consciously accepted risk Human error	25 75	10 90	15 85
Triggering components: • Site installations and excavation • Scaffolding and temporary structures • Structure • Other components	 12 9 44 35	 4 11 72 13	 13 22 48 17
If human error, then in ... • Planning and design • Execution • both of the above • Other areas	 37 35 18 10	 40 20 22 18	 20 46 20 14
If human error in planning and design, then in ... • Concept • Structural analysis • Drawings, lists, etc • Preparation of execution • Other phases	 34 34 19 9 4	 18 49 9 5 19	 15 40 8 20 17

Fig. 1/1: Results of an analysis of 800 cases of damage to structures

In many cases an insufficient consideration of damaging influences triggers the failure. Influences are overlooked, inadequately considered or ignored. Looking at the respective cases a certain amount of damage was consciously adopted as an acceptable risk. This is, in and of itself, acceptable. Construction without consciously accepting certain risks is impossible. But that 15% of the deaths and injuries come under this category gives food for thought.

Influences are neglected especially during the planning and execution phases. To improve matters here is obviously a task for the experts.

When the planning phase is considered in detail – something of direct concern to the engineer – it is found that faults in the conception of a structure as well as in analysis and dimensioning are the main cause for the number of cases of damage and also for the greater part of the damage sum and more than half of the casualties. Therefore, it would be worthwhile to devote more attention to conception, analysis, and dimensioning.

Thoroughly investigating the actual *causes*, it becomes clear that failures can be traced back to the following:

37 % ignorance, carelessness and negligence,
27 % insufficient knowledge,
14 % underestimating influences,
10 % forgetfulness, errors and mistakes,

6 % unjustifiably trusting in others,
6 % objectively unknown influences.

Further, the study showed that a considerable part of the errors could have been detected in time:

- 32 % by a careful review of the documents by the next person in the process,
- 55 % by additional checks, if one had only adopted the right strategies.

In the first case the importance of good co-operation between all those involved in the construction process is recognised, and in the second the need for well-planned quality assurance procedures. Finally, the study concludes that 13% of all errors could not possibly have been detected in advance.

Also other studies, including recent ones, show more or less similar results. Reference is made to *Pugsley, 1968; Ligtenberg, 1969; Taylor, 1975; Moffat,1976; Fraczek, 1979; Allen, 1979; Hauser, 1979; Yam et al., 1980; Melchers, 1983; Hadripiono, 1985; Ellingwood, 1987; Eldukair & Ayyub, 1991; DNV Technica, 1992; Imam & Chrissanthopoulos, 2009.*

These results should challenge engineers to give more thought to this rather unsatisfactory situation. Engineers should be motivated to learn the lessons and to draw the necessary conclusions and, finally, to let their knowledge flow back into engineering practice.

1.22 Hazard potential, safety, residual risks

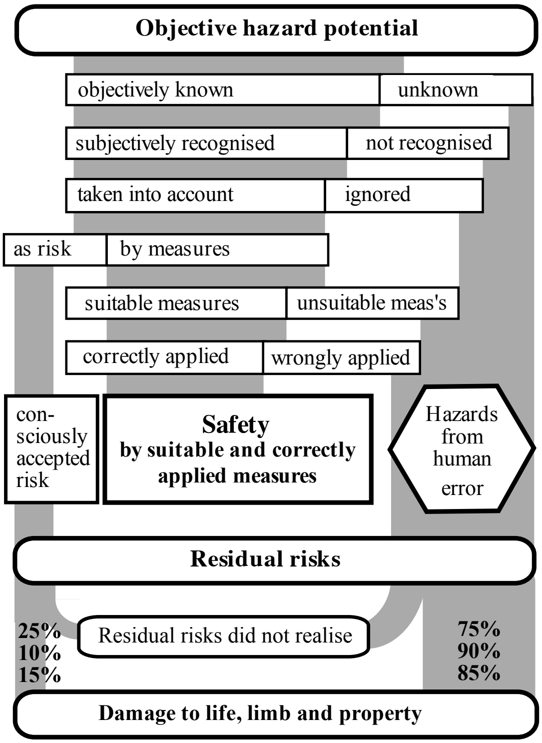


Fig. 1/2: From objective hazards to safety measures

Fig. 1/2 illustrates, in greater abstraction, the results described above. Each situation considered contains an objective hazard potential, which of course will never be completely known. It is impossible to recognise all hazards. Some hazards come as a surprise and are recognised only after they realised. This fact is known as the *Black Swan Theory (Taleb, 2012)*. A good example is the *Tacoma Narrows Bridge* disaster in 1940. Now, in 2016, the risk from aerodynamic instability is quite well known, at least for bridge engineers. Or, earlier, in 1891, the *Railway Bridge* disaster in Muenchenstein, Switzerland: a steel truss bridge failing under a fully loaded passenger train killed 70 people and initiated the well known large steel and timber column test series by L. von Tetmajer in order to finally learn that the Euler buckling formula is only valid for slender columns.

Clearly: only that part of the objective hazard potential can be investigated which has already appeared somewhere or other and is thus objectively known. In view of these, just two possible courses of action are feasible: either consciously accepting a hazard or trying – within the limits of the safety goals – to counteract it by suitable measures. In fact, however, there is a third category: In hazard recognition, as also in taking the necessary measures, errors by those involved cannot be completely ruled out. These constitute the so-called residual risks, with which engineers and the community at large have to live – whether they like it or not – and against which at the same time fight with all the means at disposal is mandatory.

Fig. 1/2 shows that in the area of hazard recognition objectively unknown hazards make the first contribution to the residual risks. A second contribution results from those hazards which – although objectively known – quite possibly can remain subjectively unrecognised. Whatever is ignored – for whatever cause – constitutes a third factor.

Hazards may be consciously accepted as acceptable risk or be counteracted by means of safety measures. Unsuitable measures or wrongly applying otherwise suitable measures then add two further contributions to the residual risks. Consciously accepted risks, together with the various contributions shown on the right of fig. 1/2 to be attributed to human error, constitute the so-called residual risks, which can never be reduced to zero.

In many cases, however, luck is around the corner, as not all of the residual risks materialise and lead to collapse, damage to life, limb, and property.

However, the fact that human errors are the cause of 75% of the number of cases, 90% of the damage sum, and 85% of all casualties, should especially draw our attention and commit us to fighting the right side of fig. 1/2.

1.23 "Snow" – an example

In many countries snow is without doubt an objectively recognised hazard for structures. Subjectively unknown are possibly the existence of some specifically snow-prone regions, as well as the possibilities of snow drifts due to wind, the formation of piled-up snow on the roofs of buildings, etc. The first contribution to residual risks comes from not recognising or neglecting such effects.

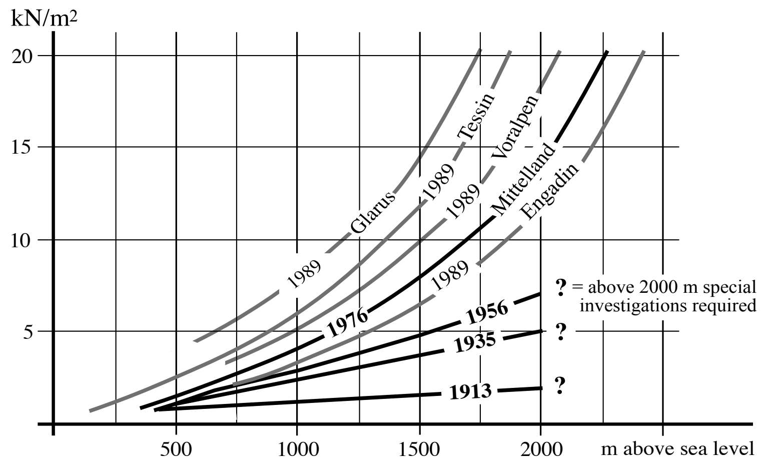


Fig. 1/3: Increase of codified snow load from 1913 to 1989 in Switzerland

Snow load values in codes certainly do not represent the maximum possible snow loads. Given values, together with the load factor, result in well-defined design values, which correspond perhaps to a one-hundred year snowfall. One obviously accepts tacitly the fact that a one-thousand year snowfall could lead to damage. But the dimensioning of all structures for such high values is regarded as unreasonable and uneconomic. Some risk is consciously accepted.

It is interesting to study the development of a standard for snow loading over the years. The results of comparing, e.g., the Swiss standard "Actions on structures" *(SIA 261, 2014)* in its various editions since 1913 (then under SIA 160) are shown in fig. 1/3.

The figure shows that although the standard on snow loading always took into account the altitude of the structure above sea level, its numerical value was increased with each successive revision. The revision of SIA 160 in 1989 introduced explicitly for the first time a regionalisation with respect to snow loading (which in the earlier editions was merely described verbally). Apparently, the willingness to accept damage to structures due to snow loading was decreasing with time, placing instead ever higher demands on the dimensioning of roof structures.

Finally, it is appropriate in connection with this example to shortly discuss the question of adequate *safety measures*:

- A practicable measure is certainly the adequate design of the structural system.
- It may not be practicable to think of clearing the snow from a glass roof in order to prevent the collapse of the structure, because the person clearing the snow might break through the glass and get hurt or even killed.
- In the case of a greenhouse, however, a possibility would also be to heat the glass roof from the inside in order to melt the snow continuously as it falls. Some risk would be involved with this measure, however, if in emergency cases like power cuts or a defective burner an adequately maintained emergency heating system is not ready to take over.

1.24 Human error

From fig. 1/1 and 1/2 it can be seen that human errors are clearly the main source of damage. Human errors fall into many categories: errors of judgement, overlooked aspects, insufficient knowledge, lack of know-how, lack of insight, incorrect or no action taken. Such errors can be combated at many possible levels, e.g.:

- *objectively unknown hazards* – by furthering fundamental research, by careful evaluation of experience and a thorough investigation of "unintelligible" phenomena (e.g., the Tacoma Narrows Bridge failure in 1943: the problem of the previously unknown phenomenon of aerodynamic instability of large span suspension bridges was discovered),
- *subjectively unrecognised hazards* – by improving basic education and training, by furthering life-long education at all levels, and by publishing examples of bad experiences in detail,
- *ignored hazards* – by clear allocation of responsibility and competence as well as by rigorously combating all forms of carelessness, negligence and ignorance at all levels,
- *unsuitable measures* – by improving expert knowledge, carefulness and overview with all those who plan the measures,
- *improper use of measures* – by requiring clear and unambiguous plans, basic documents and instruction, as well as by creating and maintaining effective control mechanisms.

All these strategies together will not of course totally eliminate residual risk. The objective hazard potential is boundless and people are inherently prone to error, since their knowledge, know-how, ability to learn, insight, and comprehension are limited, and nobody is free from error. This should not stop us, however, from trying to reduce residual risks wherever possible. Attempts are usually undertaken under the terms *Quality Assurance* or *Quality Management* (see section 5.5).

The following part of this introduction has much in common with such risk-reduction attempts in that it aims to deal with hazards in a conscious way and to introduce some order into the basis of design, execution and maintenance of structures.

1.3 Tools and Strategies

1.31 Hazard recognition

Structural safety has very much to do with the recognition of possible hazards. The goal (and also the main problem) is to recognise all possible hazards. Only then can a safe and reliable solution be found. Although this goal cannot ultimately be reached, utmost endeavour to attain it is requested.

Hazard recognition requires the engineer to exercise imagination and creativity. The main task of the engineer can be seen here, because once the potential hazards have been recognised, reducing their harmful effects is usually relatively easy. Not having recognised a hazard is, in hindsight, one of the worst experiences of an engineer.

Every structure and every situation is exposed to a variety of hazards from the natural and man-made environments and from human activity and intervention. Hazards from the *natural environment* include wind, snow, avalanches, rockfall, landslides, lightning, chemical and physical attacks, soil and ground water effects. Hazards from *human activity* include utilisation, fire, explosion, chemical and physical attacks, etc. and – primarily or accompanying – *human errors* stemming from mistakes, ignorance, and negligence. Finally, also the weaknesses of the structure itself (such as buckling, cracking, fatigue, corrosion, etc) may be considered as a type of hazard.

Certain creative techniques and cognitive aids are helpful in trying to recognise all possible hazards. Such techniques are briefly discussed below:

- *Chronological analysis:* Step-by-step the process (what, where, when will occur) is put together beforehand. Everybody applies this strategy intuitively in daily life. It is, however, also extremely useful in planning technical activities.
- *Utilisation analysis:* It is essential to analyse in advance the way the building will be used. What will affect a situation, what events will accumulate? What facilities, machines and equipment are planned? What could go wrong in the planned operations? What could break down and thus become hazardous?
- *Influence analysis:* Which quantities influence the problem at hand? One can have in mind the damaging influences in human activities and shortcomings while also looking for influences from the natural environment. Often, new situations have to be considered, which could make previously harmless influences dangerous. And, finally, components of a situation that alone are not hazardous can in combination become hazardous.
- *Energy analysis:* One can investigate energy potentials. Where could gravity; pressures; kinetic, chemical and thermal energies; electricity; electrical and electromagnetic fields; ionising rays; etc. occur in a hazardous way? Often, the failure of the supply of certain forms of energy can likewise become a danger.
- *Material analysis:* In looking for possible hazards one can consider the properties of building materials and operating systems, the use of raw materials, intermediate and end products. One can look at combustibility, explosiveness, toxicity, corrosion – also in combination.

Such strategies of thinking are applied in practice under various names and methods, e.g., Hazard Identification Study (HAZID), Hazard and Operability Study (HAZOP), What-if Analyses, Failure Mode and Effect Analysis (FMEA), etc. For further information see, f.i., *Hyatt, 2003; Ostrom & Wilhelmsen, 2012; Stamatis, 2014*.

In addition there are other methodologies which highlight certain critical situations:

- Examining interfaces: It is often fruitful to seek hazards where materials, information or responsibilities are handed over to someone else or where main functions have to be fulfilled. "What happens, if... fails?" is often a good question. The question "Why?" can often be left open, thus shortening the analysis of safety and reliability problems.
- Working with logic trees: e.g., fault trees, event trees (see section 1.33). In this way logic, order, clarity, consistency, and completeness are introduced into our thinking.

Often it is appropriate to organise *Brainstorming* Sessions searching for possible hazards and counteracting measures. In order to be really productive, the use of the word *„but"* should be forbidden in such exercises.

Finally, in searching for possible hazards one should consciously make use of experience by the act of listening and by referring to relevant literature. A rich source of experience is to be found in the codes, regulations, guidelines and recommendations of professional bodies from within a country or region (see also section 1.5).

1.32 Lessons learned from *Ronan Point*

Ronan Point (see *Ronan Point)* was a 22-storey residential tower block in Newham, East London, constructed using precast elements, walls and slabs. Early in the morning on 16 May 1968, one of the inhabitants, a landlady, went into her kitchen in a corner flat on the 18th floor of the building, and lit a match to light the stove for a cup of tea. The match, however, sparked a gas explosion that blew out the load-bearing flank walls, removing the structural support to the flat above. Leaving the floors above unsupported caused a progressive collapse of the south-east corner of the building. The landlady survived, but four other people were killed: lighting a match with absolutely disproportionate consequences.

It is quite sure that the weakness of the structure is to be seen in the poor joints connecting the vertical walls to the floor slabs relying on gravity and friction, only. Weakness of the joints contributed to the collapse, and certainly also tolerances going the wrong way. However, weakness of the joints was *not the cause* of the collapse. And neither the gas nor the match.

It clearly was *Human Error,* as it turned out that the landlady forgot to firmly close the gas valve the evening before. Gas could disappear from the gas tank, filled the kitchen, just waiting for a spark to explode. The *consequences* of this error obviously were damage to the tower and to its content, or more generally *damage to property*. And there were *fatalities*, people injured and killed.

However, a later analysis of the construction showed that not only a gas explosion could trigger the collapse but also a *strong wind*. This finding results in the conclusion that the engineer designing the joints made a mistake in not checking the joints against wind. Human Error again.

Learning the lesson: the partial collapse of Ronan Point led to major changes in building regulations all over the world. In almost any country's structural codes since clauses have been introduced

Pictures from the Internet

requiring design and execution of structures such that consequences of errors will *not be disproportionate* to the causes.

Such clauses require that ...

„Buildings shall be constructed so that in the event of an accident it will not suffer collapse and damage to property, life and limb to an extent disproportionate to the cause“.

The keyword to all of this is *Structural Robustness*. The term entered the field of structural engineering shortly after the Ronan Point accident. It asks for a senior and mentally broad approach to everything that may come to your sight and mind when inspecting drawings, fabrication and erection processes, utilisation scenarios, etc. A robust structure, or a robust situation is a state that allows for small deviations from planned states and even some unrecognised and ignored hazards (see fig. 1/2).

There have been several attempts to find quantitative ways to robustness (see also *ISO/FDIS 2394, Annex F, 2015),* none being entirely successful. Robustness remains scattered and ambiguous, making it difficult to apply generic rules and criteria to specific cases (*Knoll & Vogel, 2009)*. There is still room for qualitative opinions and judgements of experienced engineers and/or architects.

Related to robustness, but not the same, is the notion of *resilience*. Resilience is the ability to limit the damage in time and restore within a short period of time the primary functionality of a structure or system. Robustness of course already helps that way, but repair time is another important issue. Resilience is important for networks (transport, water supply, electricity, communication, so-called life lines) but also to special buildings like hospitals, police and fire brigade stations, administration headquarters, and so on. For more information see, e.g., *Ellingwood, 2007; Canisius, 2011; Main*, *2016; McAllister, 2016; Kröger, 2017*.

1.33 Working with Logic Trees

In order to study the logics between causes on their way to consequences traditionally so-called Event Trees and Fault Trees are commonly used. Bayesian Networks are more recently developed tools which should be mentioned here (see *Bedford & Cooke, 2003).*

a) Event Trees

An *Event Tree* starts with some potentially *initiating event* (IE) and presents – comparable with a trunk developing into branches, arms, twigs and leaves – all relevant possible subsequent scenarios. It may be drawn in a vertical as well as a horizontal way, the latter one being the most common. The horizontal direction, in that case, often corresponds with a chronological order of events, starting with the initiating event and ending with final consequences per scenario. An event tree is a form of so called forward logic. The aim: Inductive identification of all possible consequences, i.e., fatalities and damage to property including respective probabilities. These are calculated by multiplying all probabilities along a branch ending in a consequence.

b) Fault Trees

A *Fault Tree* on the other hand is based on backwards logic. It starts with some *undesired final event* (TOP) and shows – root like moving downward – possible causes leading to this event. The probability of the TOP event is calculated by estimating the probabilities of the basic events at the bottom. Going upward through the various AND-, OR- and other types of gates.

- An *AND-Gate* bundles the various ingoing components in the sense that all must simultaneously fail. The probability of the outgoing branch thus is calculated as the product of the probabilities of the incoming components. Therefore the „ · " sign in the symbol of the gate.
- An *OR-Gate* bundles the various incoming components in the sense that one component failing results in the failure of the outgoing branch. Thus, the probability of the outgoing branch for exclusive events is calculated as the sum of the incoming probabilities. Therefore the „ + " sign in the symbol of the gate. For independent events the probability of the outgoing branch is the sum minus the product of the incoming probabilities.

A fault tree must be oriented towards the TOP. Circular routes are not allowed. One should be careful to consider possible dependencies (correlations) between the various events. The aim is deductive identification of all possible causes of the TOP event and respective probabilities.

c) Bayesian Networks

A *Bayesian Network (BN)* simply shows events (or variables) in nodes which, if they are related, are connected by lines. For each event or variable there is a so-called logic table showing the probabilities of the various outcomes conditional on the outcomes of the related events (variables).

d) An example in three views

A simple example using respectively an *event tree*, a *fault tree* and a *Bayesian network* will be discussed. The example concerns the occurrence of a fire. Fire depends on the occurrence of an ignition (i), failure of the extinguishing system (e) and the fire load intensity expressed as the amount of combustible material (Q). The final event is a fully developed fire (FF).

In this example a bar on the symbols i, e and FF indicates the denial (no ignition, proper working of the extinguisher, no fully developed fire). For Q the presence of three levels: Q_1, Q_2 and Q_3 is defined.

For a numerical evaluation the (annual) ignition frequency is taken as 0.01, the failure probability of the extinguisher as 0.1, and the probabilities P of the fire load distribution as:

$P(Q = Q_1) = 0.3$
$P(Q = Q_2) = 0.5$
$P(Q = Q_3) = 0.2$

All numbers are fully fictitious and intended only in view of illustration.

Consider first the *event tree* (see fig. 1/4). It starts at the left side with the initiating event IE of ignition (i). If there is no ignition there is no fire (lower branch).

Once a fire has started (upper branch), the fire may be stopped by the automatic extinguishing system. This system may work properly or not. It is assumed that no fully developed fire will occur for the lowest level of fire load.

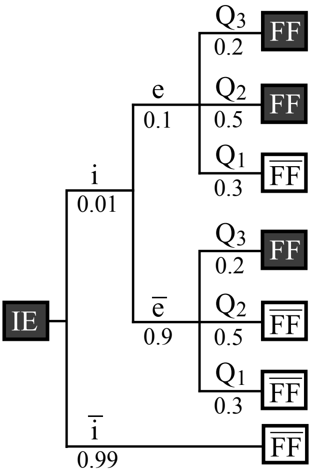


Fig. 1/4: Event tree

On the other hand, fire will always occur for the highest level regardless of the operation of the extinguishing system. For the medium class of fire load the development of a fully developed fire depend on the success or failure of the sprinkler system. This way the tree ends on the right side with seven branches in total: three branches result in fire and four do not. We may calculate now for instance the probability for each of the branches. The most upper branch has the following probability per year:

$P(1) = P(i){\cdot}P(e){\cdot}P(Q_3) = 0.01{\cdot}0.1{\cdot}0.2 = 0.0002$ (per year)

Similar calculations can be made for the other branches. Adding up the three branches leading to fire leads to the total annual probability of a fully developed fire:

$$\begin{aligned} P(FF) &= P(1) + P(2) + P(5) \\ &= 0.01{\cdot}0.1{\cdot}0.2 + 0.01{\cdot}0.1{\cdot}0.5 + 0.01{\cdot}0.9{\cdot}0.2 \\ &= 0.0002 + 0.0005 + 0.0018 = 0.0025. \end{aligned}$$

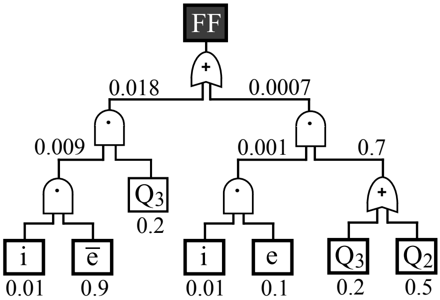
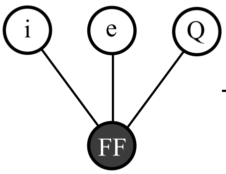


Fig. 1/5: Fault tree

A *fault tree* (see fig. 1/5) shows the fully developed fire as the undesired TOP event. The downward running branches show the various ways in which the top event may develop. These ways correspond in fact to the three branches already observed in the event tree. Note that at the AND gates one may use the standard operation for independent events (multiplication) and at the OR gates the summation (exclusive events).



i	ī
0.01	0.99

e	ē
0.1	0.9

Q_1	Q_2	Q_3
0.3	0.5	0.2

i	e	Q	FF
1	1	3	1
1	1	2	1
1	1	1	0
1	0	3	1
1	0	2	0
1	0	1	0
0	1	3	0
0	1	2	0
0	1	1	0
0	0	3	0
0	0	2	0
0	0	1	0

Fig. 1/6: Bayesian network

Finally, in fig. 1/6 the *Bayesian Network* for this case is very simple. In BN terminology there are three so-called parents (the events i, e, Q) and one so-called child (the event FF). The logic table at FF (together with the probability tables for i, e and Q) contain all necessary information.

One could even further simplify the network by dropping Q and have only i and e as parents. The information about Q is then incorporated in the following logic table for FF:

i	e	FF
1	1	0.2 + 0.5 = 0.7
1	0	0.2
0	1	0.0
0	0	0.0

Given this logic table (and the probabilities for the events i and e according to fig. 1/6) the “full fire“ probability may be calculated from:

$$P(FF) = \sum_i \sum_e P(i) \cdot P(e) \cdot P(FF \mid i,e) \qquad (1.4)$$

Numerically, this is:

P(FF) = (0.01·0.1·0.7) + (0.01·0.9·0.2) + 0 + 0 = 0.0007 + 0.0018 = 0.0025

Of course, this is the same answer as before when using event or fault trees. The interesting thing about a Bayesian Network is that it can also be used in reverse order. For instance, what is, given an observed full fire, the probability that the extinguishing devise did not work? This requires of course the right software tools.

e) Recommendations

In many applications a fault tree is used for analysing the cause of an undesired event and an event tree for the analysis of the consequences. Seeing it this way the IE of an event tree (see fig. 1/7) links with the TOP of the respective fault tree.

Event trees normally end in "F" and "P" indicating fatalities and damage to property. Fault trees often have their final roots in Human Error (HE), though most often investigations might not go that deep.

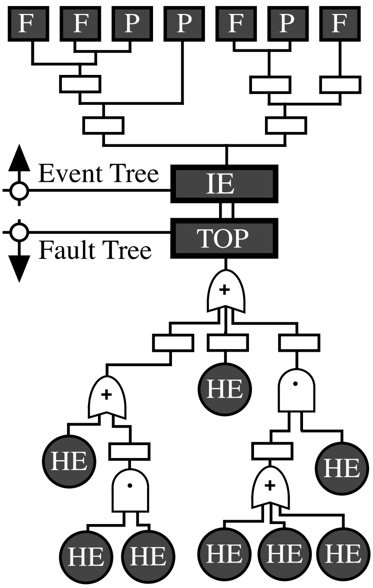


Fig. 1/7: Tree from roots to leaves

It is strongly recommended to not stop searching through logic trees without asking the important question "Why" and without carefully looking to the answers. These often indicate the path to proper and effective counteracting measures.

Central characteristics of both kinds of trees are:

- Choosing the IE and the TOP events is arbitrary in the sense of "Assume that IE and/or TOP happens!". And as within Brainstorming sessions the word *"but"* is forbidden. Only if you admit that the events may become true, working with trees is possible.
- It is useful to set IE and TOP events at different levels, i.e., closer to the causes or closer to the consequences in order to fully explore the situation.
- A graphical visualisation of such trees (in fig. 1/7 shown just symbolically) is only possible in very simple cases. Computer programs may be of help or are not avoidable at all.
- Influences of system, environment and human intervention may easily be mixed.

Trees are of good help when it comes to talking and convincing lay people about what is possible and what should be done.

1.34 Hazard scenarios

If different hazards occur together in space and time, situations can arise that may be more dangerous than those from individual hazards acting alone. In the case of structures this is the rule rather than the exception. The concept of *hazard scenario* (see *Schneider, J., 1985)* is instrumental in dealing with this fact. Hazard scenarios describe the combined action of hazards in a way suitable to engineering thinking. Instead of single hazards a number of hazard scenarios then have to be taken into account in seeking suitable countermeasures.

A practical example will best illustrate this approach, which admittedly at first sounds rather complicated. Fig. 1/8 shows the situation before the collapse of the roof of a platform at the railway station in *Einsiedeln,* Switzerland, which took place in 1970. The depth of snow was very high and blown into an odd shape so that it produced a large bending moment in the columns. This situation by itself was accounted for in the design. The accompanying action was wind coming from an unfavourable direction. This wind now acted on a much greater area due to the geometry of the snow mass. At that time the increased area of wind attack was not considered in the design because it was not stipulated in the codes.

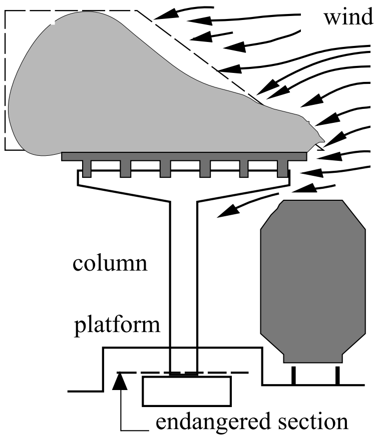


Fig. 1/8: Hazard Scenario example

The code of practice then in force for the design of the structure only provided for the superposition of the internal forces considered separately due to snow and wind. The code clauses for snow did foresee a one-sided loading. The wind forces, however, had only to be applied to the net area of the structure itself, i.e., without the increased area produced by the mass of snow.

In addition, there was a train situated in an unfavourable position from the point of view of wind flow conditions and resulting wind action. This combination of effects finally led to collapse. Fortunately, at the time nobody was under the roof because of the bad weather conditions, and so there were no casualties.

Seen historically this collapse gave birth to the *hazard scenario concept* and led to its introduction into the Swiss codes in 1989. This also led to the abolition of the term *load combination* and the departure from the former practice of combining load cases or adding up action effects from different sources in a structure.

A so-called morphological test (see *Zwicky, 1989)* can often be helpful when searching for possible hazard scenarios. In fig. 1/9, in the form of a matrix, hazards from different sources and at different points in time in the life of the structure are shown. The hazard scenario "snow" of the previous example has been entered as the *leading hazard* together with a number of possible *accompanying actions*. The column head provides the name of the hazard scenario.

In principle, every point of intersection in this scheme can represent a leading hazard and thus define a hazard scenario. The possible accompanying actions must then be sought in the corresponding row. Admittedly, not all accompanying actions are relevant. It is also obvious that not every intersection would represent a hazard scenario relevant for planning preventive measures.

■ Leading hazard

● Accompanying hazards

		Hazards due to ...												
		human activity, utilisation and man-made environment							natural environment					
		......	Utilisation	Overloading	Collisions	Explosions	Fire	......	......	Wind	Snow	Earthquake	Water	Mud flow	......
Execution phases	Excavation														
	Foundation														
	Transport														
	Erection														
	Concreting														
	Prestressing														
	...														
Utilisation phases	......														
	normal		●					●		●	■				
	special														
	......														
	Inspection														
	Maintenance														
	......														
Demolition															

Fig. 1/9: Morphological test in the search of Hazard Scenarios

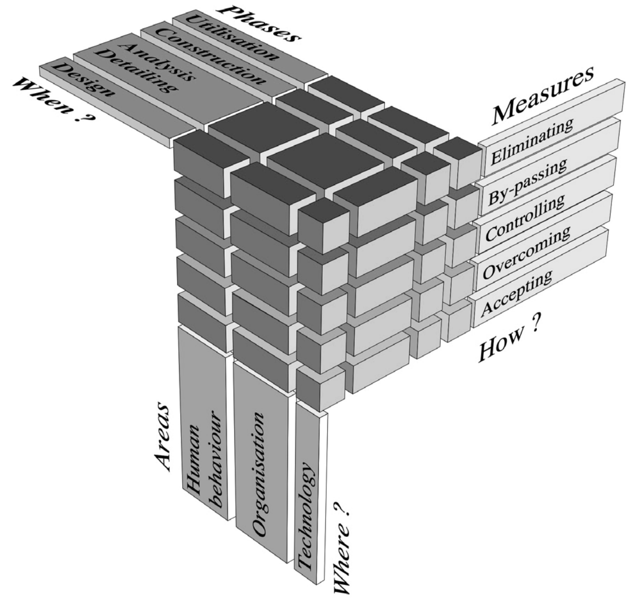
1.35 Categories of countermeasures

The hazards characterised in hazard scenarios can either be combated by applying suitable countermeasures or by consciously accepting part or all of a hazard scenario as an acceptable risk. Five categories of measures have to be distinguished. Hazards may be:

- *eliminated* by taking adequate action at the source of the hazard itself,
- *by-passed* by changing intentions or concepts,
- *controlled* by checks, warning systems, monitoring, etc.,
- *overcome* by providing sufficient reserves,
- *consciously accepted* as unavoidable or as representing an acceptably small risk.

The above measures are applied in the *technical* and in the *organisational* areas, as well as everywhere *human behaviour* can be influenced. Examples of measures are using one weld size only on a specific structural part, or organising checking procedures for weld sizes, or firing welders who do not read drawings thoroughly enough and thus keep to instructions. Furthermore, measures are applied in *all phases* of the construction process, during the design, the construction, the utilisation, and the demolition phases.

This methodology results in a three-dimensional morphological box called here the *Safety Cube* (see fig. 1/10). The cube shows the logical arrangement of all conceivable possibilities of measures that may be chosen to counteract hazard scenarios.



Areas - Where?
Technology
- Organisation
- Human behaviour

Phases - When?
- Design
- Analysis and detailing
- Construction
- Utilisation

Measures - How?
- Eliminating
- By-passing
- Controlling
- Overcoming
- Accepting

Fig. 1/10: Possibilities to counteract Hazard Scenarios – the Safety Cube

It should be observed that often use is made of safety measures only from a small part of the safety cub*e*, e.g., solely in dimensioning structural parts in analysis and design to withstand hazards. Thus, frequently design work is uneconomical and less than optimal. Optimally, preventive measures corresponding to a hazard scenario should be chosen from *all* parts of the cube.

1.4 Organisation of work

1.41 Hierarchical ordering of functions and tasks

The life of a structure starts with the first thoughts of the owner about whether a new building or structure would help to solve some of his problems, needs, or desires. He/she might also be confronted with maintenance needs of his existing building stock and needs to think about how to set priorities. In this situation he/she would be well advised to seek expert knowledge and to get in touch with professionals, with engineers and architects. In doing so, in view of the experts, he turns into the position of a client.

Within this small team in thorough discussions the needs of the client will be analysed and formulated in a manner which is helpful to organise the process from the beginning of the planning phase through the execution phase right up to the use and maintenance phases, i.e., during the complete life of a structure.

Fig. 1/11 shows, against the background of national and/or international codes and regulations, in hierarchical ordering above right the client with his requirements and above left the consultants exercising the relevant expert knowledge.

The client and the consultants are expected to meet in order to carefully prepare the most basic document leading the whole building process.

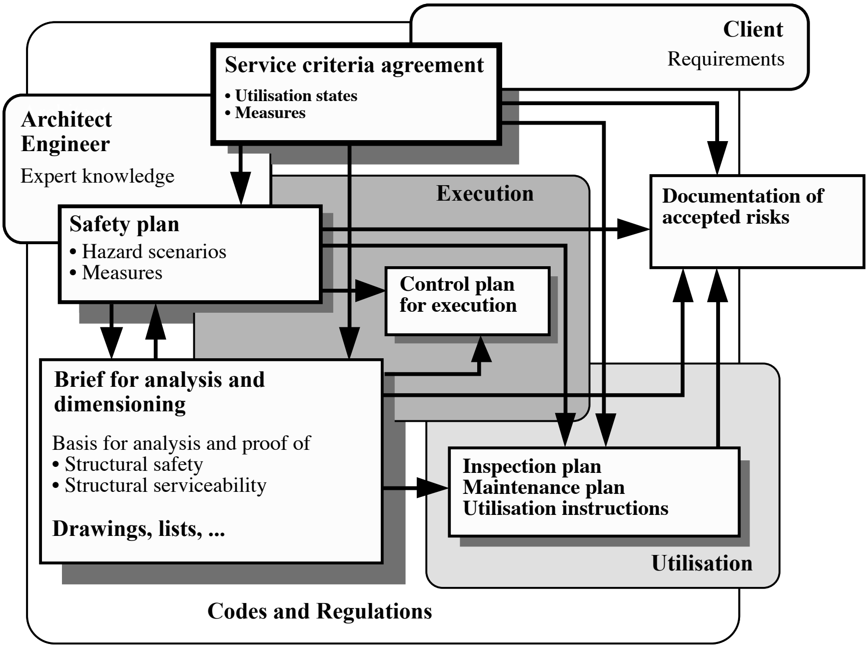


Fig. 1/11: Hierarchical ordering of tasks and documents

The team might chose different notions and titles of this and all other documents mentioned in fig. 1/11, but when it comes to see the contents it's all the same. The leading document here is called the *Service criteria agreement*. This document later serves as a basis for a document called the *Safety plan (Schneider, J., 1989)* which is prepared by the consultants under their own responsibility.

1.42 Service criteria agreement

The Service criteria agreement (or however it is called) is an extremely important document. Great importance should be attached to its preparation. The client is confronted in these discussions with the possibilities and limits of the experts and must in the end agree to the jointly formulated content of the agreement and sign the document. The discussions also give the experts a good opportunity to present their value judgements.

The Service criteria agreement contains answers to the typical question *"What is planned here?"*, or, more specifically, to questions of the following kind:

- What exactly does the client want in detail? What are his aims? To what does he attach the most importance?
- What requirements (e.g., for the behaviour of the structure in relation to, for instance, vibration, crack development, water tightness, etc.) does he stipulate?
- Are any environmental effects to be expected here?
- Does the structure influence the environment in any important way?

1.43 Safety plan

The users of structures (e.g., the occupants of a building or the car drivers on a bridge), other persons in their vicinity, and the public at large demand adequate safety of persons in the structure's area of influence. This demand is reflected in the Building Codes and other laws in each country. Safety is – in contrast to the rules in the service criteria agreement – not a matter of consultation, discussion and agreement with the client, but a central task of the experts within the framework of the general, ethical, and legal rules. The experts have to develop the safety plan on the basis of the service criteria agreement. In this task the engineer together with the other experts involved has a very responsible position.

The safety plan actually develops into a series of documents describing the relevant hazard scenarios and the corresponding countermeasures. Here too, the method is to pose relevant questions and provide appropriate answers. It is a matter of thinking and imagination on what could prove hazardous to the construction and service processes. Typical questions are:

- Which hazards are implied with the planned use?
- Which hazards could stem from the different components of the structure?
- What could go wrong in operating the structure or building?
- Which hazards threaten the structure from the environment?
- Which hazards threaten the structure due to human activities?

These hazards can be put together by creative use of methods and by using the hazard scenario concept as introduced above. In doing so, ideas emerge simultaneously about possible preventive measures. This marks the start of planning the safety measures with the typical question:

- What preventive measures can be implemented effectively?

Everything that can or has to be left as an acceptable risk, with no preventive measures, is entered into a documentation of accepted risks.

1.44 Operational documents

The measures specified in the service criteria agreement and the safety plan are best organised into separate documents for particular target groups of people involved in the process (see fig. 1/12). These documents include:

- Brief for analysis and dimensioning
- Construction inspection plan
- Inspection and maintenance plans for the service phase
- Utilisation instructions for the users

The client can contribute little to the safety plan or to the above group of documents. He can scarcely judge the correctness of their content, and for this reason he takes no responsibility. Therefore, it will not be expected that he should sign these documents. By contrast, however, the client must definitely be confronted with the documentation of accepted risks.

The requirements of the formal structuring of the service criteria agreement and the various components of the safety plan depend on the potential hazard to persons and the environment. In many simple cases of daily practice, it will be sufficient to follow the codes of the particular country dealing with design and construction. The corresponding documents, therefore, can be kept short.

The documentation of the accepted risks – a separate document – contains those risks, which – usually for economic reasons – have been studied and found to be acceptable. Usually, the client profits from the lower costs associated with accepting greater risk. Therefore, it is justified to place on his shoulders the corresponding financial risk. That is why the client has to agree to this document with his signature. If he is not prepared to sign, it may help to advise him to seek a second independent opinion. Sometimes, in order to reduce the risks to a level acceptable to the client, additional measures have to be introduced into the safety plan.

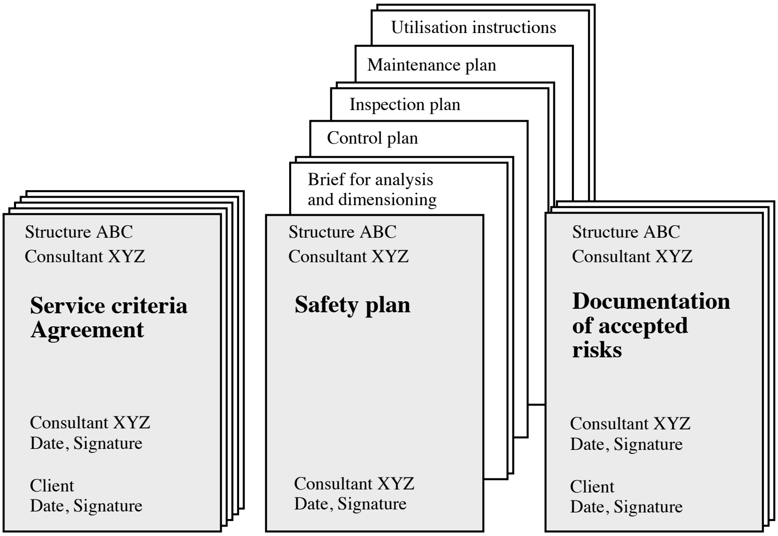


Fig. 1/12: Guiding documents from planning to execution, use and demolition of structures

Confronting the client with risks and costs is very important and is mandatory from the point of view of the duties of due care and liability of the engineer laid down in the laws of most countries.

All these documents, together with the plans of the architect, the structural engineer and those of other experts, are to be made available throughout the life of the structure, in order to have the necessary information accessible for any future alterations.

1.5 Codes and standards

1.51 Use of codes

In building and civil engineering there is a long tradition of designing and assessing structures on the basis of codes and guidelines. These documents more or less prescribe the structural calculation models, the values for the load and resistance parameters, safety margins, the way to cope with accidental and extraordinary design situation, etcetera. The intention is to arrive at structures that are both safe and economic and serve implicitly or explicitly sustainability goals like limiting emissions and consumption of natural resources (see, f.i., *Sakai, 2013)*.

The advantage of standardization is that the analysis and criteria for assessing a structure becomes less dependent on subjective opinions and/or conflicting commercial interests. The disadvantage is that engineers may become lazy and follow the codes without much reflection. Most codes, of course, make clear that this is not the intention, but that of course does not prevent from laziness.

In principle, most codes have a strong national or regional (Europe, USA, Asia, ...) character, reflecting often the tradition of the local building industry and safety culture. There is a trend for harmonisation to reduce differences that may hamper the trade on the one side and are not defendable from a scientific point of view on the other. The International Standards Organisation ISO tries to harmonize codes on an international worldwide level. In practice these codes are used seldom directly, but they serve primarily as a "code for code makers". Scientific progress is often first codified in ISO codes which then may serve as a guideline for other more practical standards.

1.52 Present day Codes of Practice in the building industry

At the top level most modern codes of practice require that structures shall be designed, executed, operated, maintained and/or even decommissioned that it will serve society for its intended functionalities. In particular it shall, with appropriate degree of reliability and in a socio-economic and sustainable way fulfil the following performance requirements:

- function adequately under all expected actions throughout their service lives;
- withstand extreme actions and environmental exposures;
- show sufficient robustness in case of unforeseen events.

Usually codes recognize the random nature of both external loads as well as the structural properties. In the practical elaboration, however, most codes only give attention to semi-probabilistic methods like the Allowable Stress Method, the Load Factor Design (LFD) approach or the Partial Factor Design. The calculation methods, values for loads and resistance as well as safety margins used in these methods are based on tradition, risk based calibration or a combination of both.

Given the increasing complexity of structural systems, the use of new concepts and materials, modern social demands with respect to safety, serviceability and sustainability it must be clear that tradition alone cannot be a solid way to safety and reliability of modern and future structures. The experience from the past is only for a part relevant for the future and, in addition, traditional methods have been developed on the basis of many arbitrary decisions and assumptions which do not have to be optimal at all. The shortcomings of for instance the allowable stress method to reach a uniform safety level were already known in the 70-ties when non-linear methods of analysis entered building engineering practice. There is a strong need to choose for a better funded theoretical background of codified rules, even if their final appearance shows come resemblance with the codes of the past.

Most codes (or laws referring to them) allow to deviate from the codes if it can be proven that structures of similar quality are obtained. Such codes are called performance based codes as opposed to codes that only allow one prescribed solution. Some codes, for that reason, mention explicitly the target reliability (as a maximum acceptable probability of failure). Every design or existing structure that meets that requirement is okay. Verification is allowed on the basis of probabilistic methods or well calibrated semi-probabilistic ones, applied models may be simple as well as advanced, numerical or experimental, however, of course, embedded in the correct reliability approach. In some cases and for some aspects even a risk optimization is recommended, leaving even free the (optimum) level of reliability for a certain aspect. Note however that these requirements may differ quite a lot from country to country.

Even if working with a code that allows or even recommends in certain cases a full probabilistic or risk based approach, there still is the difficulty to find relevant statistical data. In order to close that gap the *JCSS* has developed a code for full probabilistic design and assessment *(JCSS 2001,* with background in *Ditlevsen, 1988; Ditleven & Madsen, 1989; Vrouwenvelder, 2002)*. The code hardly gives information on calculation models for, e.g., a column under compression or a shallow foundation, but offers means, standard deviations, etc., to enable probabilistic reliability calculations in a more or less standardised way. The same code can also be used for calibration purposes when defining partial factors in semi-probabilistic design.

1.53 Standard ISO 2394 Reliability of structures

The *ISO standard 2394, 2015* (see also *Faber, 2015 and Phoon et al., 2016)* is an international code that aims at a practical but also rational and scientific approach to the design and assessment of civil engineering and building structures. The code may be applied directly but is also intended to harmonise the various national and regional codes all over the world.

ISO 2394 is systematically based on a scenario approach to performance based decision making, including risk considerations and socio-economic optimization. In its elaboration the standard provides for approaches in practical application at three levels, namely:

- Risk based design and assessment
- Reliability based design and assessment
- Semi-probabilistic design and assessment

On the highest level a full integrated risk analysis is carried out including steps like system definition, hazard analysis, counter-measures and optimization. In the reliability based approach, the analysis is narrowed to a probabilistic analysis of the structural behaviour and the verification whether certain safety targets are met. The targets themselves have been established on a quite general level outside the code. Semi-probabilistic design has the same scope as reliability based design, but is based on simplified methods using a set of so-called characteristic values and partial (safety) factors. These values and factors have been calibrated in such a way that the same safety targets are met as in the case of reliability based design.

Essentially the three methods aim at the same goal and need the same information. Due to the elaboration of the levels, the task for the engineer is more demanding on the highest, the risk based analysis, while in general results are more conservative on the lowest level, the semi-probabilistic design and assessment.

Most national codes give a full operational elaboration on the semi-probabilistic level only. For reliability and risk based procedures relevant information can be found in documents of the JCSS.

Typical recommendations in this code can be summarized as follows:

a) Interpretation of probability

Decisions concerning structures shall account for all uncertainties of relevance for their performances including inherent natural variability (aleatory uncertainty) and lack of knowledge (epistemic uncertainty). This means that also model uncertainties (due to imperfect models) and statistical uncertainties (due to limited statistical data) should explicitly be accounted for. All uncertainties shall be considered in the analysis using the theory of probability. In structural reliability analysis the Bayesian interpretation of probability should be considered as the most adequate basis for the consistent representation of uncertainties, independent of their sources.

b) Target probabilities

In setting target performance levels, the fundamental principle of the marginal life saving costs for the regulation of life safety applies and is recommended. The use of the marginal life saving principle shall ensure that the safety for people using or otherwise exposed to a structure has a certain level, such that the costs associated with saving additional lives through additional safety measures, exceed the corresponding marginal life saving costs. This principle can be seen to be coherent with the general formulation of the *ALARP* (As Low As Reasonably Practical). For details see section 5.15 of this book.

c) Approach to Human safety

To quantify the marginal life saving costs a suitable method such as the *Life Quality Index* (LQI) applies. The Life Quality Index (LQI) was proposed in the late 90's (see *Nathwani et al., 1997; Lind & Nathwany, 2012)*. The philosophy behind the LQI is that the preference of a society in regard to investments into health and life safety improvements may be described in terms of the life expectancy at birth, the Gross Domestic Product (GDP) per capita and the ratio between working time and leisure time. Based on the LQI it is possible to derive the so-called limiting value of life saving costs, i.e. the necessary and affordable expenditures that should be invested into saving one additional life. Alternative approaches from the field of economics such as contingent life value evaluations have been found to yield results similar to those of the LQI. Based on LQI well-known quantities like the *Societal Willingness To Pay* (SWTP) and the *Societal Value of a Statistical Life* (SVSL) can be assessed.

d) Quality management

In general, quality management systems for construction works shall be risk-based and according to an integral approach, encompassing human errors, design errors and execution errors. When dealing with time-dependent structural properties, the effect of the quality control and inspection and repair procedures on the probability of failure should be taken into account.

e) Models

The modelling of the performance shall address all relevant issues concerning the intended use of the structures, the safety of people as well as the qualities of the environment and economy throughout the entire lifecycle of the structure. Special considerations shall be given to the modelling of the interaction between the structure and its surroundings, dependencies between the structure and possibly present mechanical engineering systems as well as the influence of human and organisational errors. The models shall include the loss of the structural load bearing functionality and corresponding direct and indirect losses. The indirect losses are related to collapse of structural parts not damaged by the initial cause and as such related to the robustness qualities of the structure.

f) Limit state approach

The performance of a structure relates to the structure as a whole or parts of it. In order to assess the performance, the use of relevant limit states is recommended for simplification. Where relevant the influence of damage on stiffness and strength of the structural elements shall be considered. Degradation of material and structural properties may be caused by energy related to mechanical, chemical physical and biological influences.

Many of the issues mentioned above in relation to the ISO 2394 will be discussed in more detail in the remaining chapters of this book.

2. Information Processing

2.1 Elementary probability concepts

Probability analysis is of great importance in dealing with risks. Although this book is not the right place to give a thorough introduction to probability theory, it is nevertheless necessary to summarise the most important concepts and principles. A more detailed discussion can be found in the literature (see, e.g., *Bernardo & Smith, 1994; Castillo, 1988; Lindley, 1965; Miller, 1990; O'Hagan, 1994; Papoulis, 1965; Tribus et al., 1969)*.

2.11 Events

The following are examples of what is meant by the term *event:* ... a pump is not working properly within a given period, ... the wind velocity within a given period exceeds a certain value; ... the steel strength of a pipe is below a given value; ... a hazard is detected in good time; ... the result of a series of chemical reactions (e.g., a chain reaction); ... a person dies in a traffic accident; etc.

An event that cannot be subdivided is called an *elementary event E*. The space in which a particular event occurs is called the *event space Ω* (fig. 2/1).

The symbol A∪B describes the union of the events A and B, represented by the shaded area. The corresponding verbal expression is "A and/or B". Fig. 2/1 demonstrates this in a so-called Venn-diagram (John Venn, an English logician, 1834 – 1923).

By the symbol A∩B the intersection of the events A and B is meant, again represented by the shaded area. The corresponding verbal expression is "both A and B" (Fig. 2/1).

Events can be independent of one other: e.g., A = not reaching a specified steel strength in a pipe and B = the fact that the pipe is painted blue. Events can, of course, also depend on one other: e.g., A = not reaching a specified steel strength and B = the fact that the pipe has burst. Thus, attention has to be paid to possible dependencies.

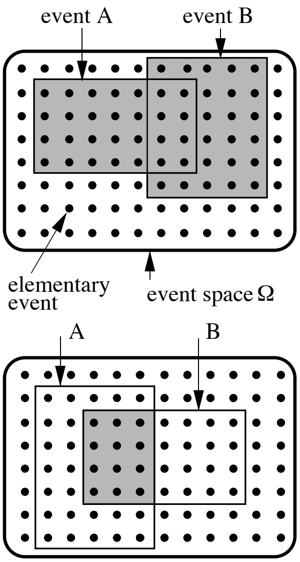


Fig. 2/1: Venn diagrams

2.12 Probabilities

a) Classical probability

Classical probability is given by the simple relationship:

$$p = \frac{\text{number of successful elementary events}}{\text{number of possible elementary events}} \tag{2.1}$$

Those elementary events are successful which fulfil a given criterion. In the case of throwing a dice, for example, for a sufficient number of throws (theoretically an infinite number), the value of "number of five's divided by number of throws" always approaches with ever greater accuracy the value 1/6. Basically the probability is 1/6 for every throw that any one of the six possibilities results (e.g., a five). The conventional way of writing this is:

$$p(\text{result} = 5) = \frac{1}{6}$$

This probability is in effect a property of an *ideal* dice. A *real* dice, may, in fact, show significant deviations from the above probability. This difference can, however, only be seen after a large number of throws. But, whatever the difference, the probability attached to e.g., a five, is an *inherent property* of the dice.

b) Empirical probability

Empirical probability is derived from the fact that one can measure a property X of an object under consideration, e.g., the life of electric bulbs. Given one or more sufficiently large samples, one can derive from these what relative frequency of failure an electric bulb has on average during a given period of time. One writes this as:

f(failure of electric bulb) = 0.30/year

In section 1.1 it was already mentioned that it is wrong to speak of probabilities of failure or occurrence as long as the numbers have dimensions.

In an analogous way one can also measure the compressive strengths of a large number of concrete cylinders and obtain from the results the values of, e.g., mean, standard deviation, coefficient of variation, and sometimes further attributes of the sample. Section 2.2 will go into greater detail.

c) Subjective probability

Subjective probability is tied to everyday speech, e.g., such as "the probability it will rain tomorrow is 30%". In written form this is:

p(tomorrow it will rain) = 0.30.

Everybody knows, of course, that it may or may not rain tomorrow, i.e., that the correctness of the statement is either 100% or 0%. It cannot rain just 30%. Nevertheless, the statement can under some circumstances be useful, e.g., regarding the question whether to carry an umbrella. With new information (e.g., from looking at the sky when leaving the house), however, the degree of belief that it will rain changes.

In the same way a statement of the following type can be useful: The probability that this bridge will fail if a particular vehicle passes over it is 5%. Here, it is clear that during the crossing by this specific vehicle the bridge will without doubt either collapse or not collapse. There are only the two possibilities. It cannot collapse by 5%. Nevertheless, the statement about the probability of failure in relation to the question, say, of "should one keep the bridge open to traffic" is in some circumstances of great importance.

That such a subjective probability is not an inherent property of the bridge, but someone's opinion regarding the safety of the bridge (which by the way can change if additional information becomes available), should be clear. The person in question has, of course, to be technically competent if the statement is to have any value.

2.13 Axioms and computational rules

In the present text several axioms, terms, and notations are employed, which are very briefly summarised here. The terms originate essentially from the vocabulary of classical and empirical probability theories. They are, however, also valid in the area of subjective probability.

Probability analysis is founded on the *axioms of Kolmogoroff* (1933). These are:

- A probability is dimensionless and can only have values between zero and one.
  $$0 \le p(A) \le 1 \tag{2.2}$$
- A certain event has a probability of occurrence of one.
  $$p(S) = 1 \tag{2.3}$$
- If the intersection of A and B is the null set, i.e., if A and B do not possess any common elements, then the probability of the union of both events is equal to the sum of the probabilities of the individual events
  $$A \cap B = \varnothing \Leftrightarrow p(A \cup B) = p(A) + p(B) \tag{2.4}$$

The following computational rules apply. No proofs are given.

$$p(\varnothing) = 0 \tag{2.5}$$

$$p(A \cup B) = p(A) + p(B) - p(A \cap B) \tag{2.6}$$

The *conditional* probability (read as "probability for A, given that B has occurred") is:

$$p(A \mid B) = \frac{p(A \cap B)}{p(B)} \tag{2.7}$$

If A and B are independent events, then:

$$p(A|B) = p(A) \tag{2.8}$$

$$p(A \cap B) = p(A) \cdot p(B) \tag{2.9}$$

For some of the examples the theorem of total probability may be helpful:

$$p(A) = p(A \cap B) + p(A \cap \mathrm{not}B) = p(A \mid B) \cdot p(B) + p(A \mid \mathrm{not}B) \cdot p(\mathrm{not}B) \tag{2.10}$$

2.14 Determination of probabilities

a) Subjective probabilities

Subjective probabilities, i.e., degrees of confidence in a statement, are often spontaneously expressed, even by people who are not familiar with the methods of probability analysis.

By means of suitable questions one can check such spontaneous statements and express them in numbers. This process of asking questions can be formalised – e.g., allowing the questioned person to *choose* between the game *"All or Nothing"* and a *lottery*. In the game, e.g.:

- G_0 = 100 Euro if the statement A under consideration is true, otherwise nothing,

while the lottery

- has an average value of expectancy of G,

whereby, of course, $G \le G_0$.

The choice is usually easy to make. By varying G, however, a value can be found for which the person questioned is no longer able to decide, so that the game and the lottery appear equally attractive to him. For this specific value of G the following expression is valid:

$P(A|I) = G/G_0 = p.$

The value p corresponds to the degree of confidence of the questioned person regarding the statement A under discussion. That this depends on the amount of information I available to the questioned person, is expressed by the corresponding conditional probability P(A| I). Typically, such probabilities are estimated differently by each person questioned, i.e., they are subjective.

As a side remark: many people exhibit some risk aversion and would prefer a certain profit of say 46 Euro above a 50% bet on 100. So, p = 0.5 would correspond to $G/G_0 = 0.46$. Numbers may become different at higher amounts of money.

Subjective probabilities can also be obtained by questioning a group of experts. At an international conference on "Timber in Structural Engineering", quite central in the conference premises a test facility was positioned. The facility showed a wooden beam, 6 m long, its section being 120x200 mm^2, prepared to be loaded by a single load at mid-span. Testing this beam was announced for the end of the conference. Meanwhile all the experts present were asked to visually inspect the beam and to finally predict by ballot vote the carrying capacity, i.e., the load r under which the beam would give up and fail under either bending or shear.

Almost all of the people present were able to calculate the design strength, guided by their country's structural code provisions. But guessing about the real bearing capacity of the beam seemed to be difficult. The participants were asked to give their estimate of r (up to 1 kN precision) and put their ballot in the ballot-box. The person predicting r closest to the final test was promised a prize.

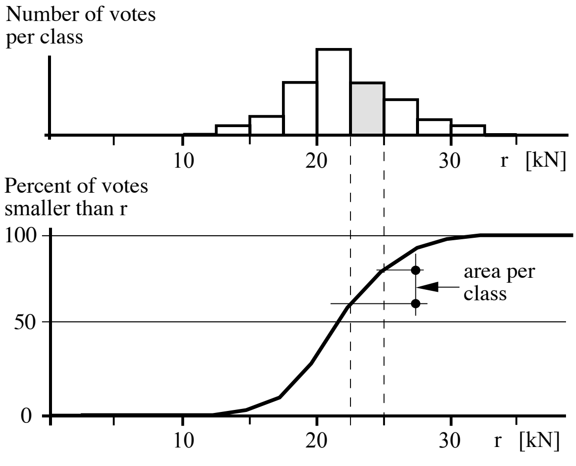


Fig. 2/2: Opinion of experts about load carrying capacity of a timber beam

The organisers analysed the replies and plotted these on the strength scale. The two diagrams shown in fig. 2/2 are obtained.

The information obtained in this way on the load carrying capacity of the timber beam is, obviously, an expression of the opinion of the experts present and not of the load carrying capacity itself. The latter could be determined only by carrying out a test resulting in a quite definite numerical value.

b) Empirical probabilities

The determination of empirical probabilities is clearly based on the *observation* of properties of interest. As an example, consider the observations of the compressive strength of concrete cylinders over a whole year. From the results so obtained one can then find the mean, the standard deviation, and further parameters of the sample and, e.g., also the probability that the strength X is not less than a given value x_0.

Empirical probabilities are thus derived using descriptive statistics. This is discussed in more detail under section 2.2.

c) Bayes updating

Subjective probabilities can be combined with empirical data. With reference to the timber beam discussed earlier, suppose that the experts were informed that a beam of the same dimensions had been tested and failed at 18 kN. Given that piece of information, the experts were asked to reconsider their estimate. Some did, some did not. If the uncertainty in the outcome of the test is considered as purely aleatory, there is no argument to change. If on the other hand the *uncertainty* is of an epistemic (lack of knowledge) type of, e.g., the tested beam of the type same type of wood, the outcome may give rise to adjustment of the estimate in the direction of the "similar beam" result.

That opinions change with information obtained is closely associated with the name *Thomas Bayes*, an English mathematician, and the Bayes' Rule or Theorem (1763). Starting from a-priori probabilities (drawn from experience, or estimated) additional information may be incorporated using Bayes' Theorem thus arriving at a-posteriori probabilities that are proposed to be better estimates. The Bayes' Theorem is thus a mathematical expression of the characteristic properties of subjective probabilities and forms the bridge between the subjective and the classical and empirical probability theories.

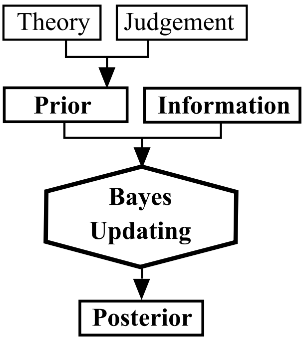


Fig. 2/3: Bayesian updating

Bayes' Theorem is shown in the following formula:

$$P(B_i|I) = \frac{P(I|B_i)\cdot P(B_i)}{\sum_{i=1}^{n}\left[P(I|B_i)\cdot P(B_i)\right]} \tag{2.11}$$

This holds for any combination of events I and B_i. Let I be the outcome of the beam capacity discussed under b) and B_i the outcome of a test on a similar beam, then the use of Bayes' theorem is the way to calculate the corresponding changes in the probability distribution.

Details on the application of Bayes' Theorem are discussed in section 6.2.

By the way: close to the end of the conference, the beam treated above was tested. The test result was much higher than expected by most of the participants. The beam resisted with much noise until 27 kN and gave up in bending mode by tensile cracking of the lower fibers.

2.2 Evaluation of samples

2.21 Basic notions

In statistics, the totality of possible observations or tests under the same conditions is called a *population* and each individual test or each individual observation is an *element* of this population. This element can be investigated with respect to different *properties* which can be treated as a random quantity or a *random variable*. Always, in statistical investigations, one is able to consider only a subset of the elements of the population. This subset is called a *random sample* and the number *n* of the elements contained in it is called the *size* of the sample.

The concrete in a building, e.g., is a population. Each arbitrarily selected zone is an element of this population. Observations are necessarily limited to a number of cylindrical specimens drilled out randomly from the concrete. This is the random sample. The observed property is, for example, the compressive strength of the cylindrical specimen.

The scale of all values is denoted by the random variable, which can be a quantity like the compressive strength of concrete. Random variables in this book are in general written in capital letters: compressive strength X, or height H. Individual values, for instance individual measurements – the so-called realisations of random variables – are written in small letters. An index can be added: the third measurement e.g., was: $x_3 = 34.6\ \text{N/mm}^2$.

A random sample, depending on the problem at hand, can have a size of three, ten, or even several hundred observations. The greater the number of samples that are available, the greater is the confidence in respective statements. But at the same time the effort also increases. After a certain number of observations the effort of taking further observations and evaluating them only increases, while the accuracy of the parameters derived from the results hardly increases at all.

2.22 Histograms

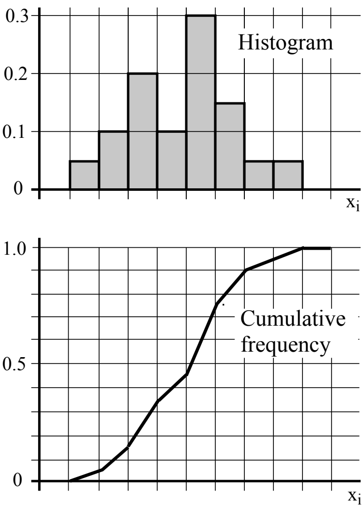


Fig. 2/4: Presentation of a sample

The results drawn from a sample are best presented graphically as a so-called histogram, shortly introduced already just a few pages ago. Above each class, the corresponding number of values per class, or still more generally the relative frequency, is plotted.

Even more insight is achieved by presenting the cumulative frequency. Here, beginning on the left, the frequencies are plotted continuously and the points are connected linearly with each other: A frequency polygon (fig. 2/4) results. This is, of course, simply the integration of the histogram.

Not only the transfer from data into histograms but also many other tasks encountered when analysing data are quite easily solved using statistic software widely available in the internet (see *StatSoftware, 2016)*.

But having a suitable program is not enough. Don't be too quick. An experienced engineer will carefully check the data set for consistency before trusting computer programs.

2.23 Parameters from random samples

A series of values – i.e., a random sample of the size n (n = number of elements of the sample) and the associated histogram – can be described by a few characteristic numerical values, so-called *parameters*. Parameters may, alternatively, be replaced by so-called *moments.* The main moments of a sample are the *mean,* the *variance* (resp. the standard deviation), the *skewness,* and the *kurtosis*. Usually the first two moments suffice.

The first moment of a random sample, the *arithmetic mean* m_X, is calculated as follows:

$$m_X = \frac{1}{n} \cdot \sum_{i=1}^{n} x_i \tag{2.12}$$

As a measure of the scatter the second central moment, the so-called *variance* of X, abbreviated to Var(X), is determined as follows:

$$\mathrm{Var}(X) = s_X^2 = \frac{1}{n-1} \cdot \sum_{i=1}^{n} (x_i - m_X)^2 \tag{2.13}$$

In some cases "n" is used instead of "n–1" before the summation sign. For small values of n this can lead to significantly biased results. Using "n–1" removes the bias. In the process of moving from the data to reliability theory, however, these differences normally disappear.

The so-called *standard deviation* s_X is the square root of the variance, i.e.:

$$s_X = \sqrt{\mathrm{Var}(X)} \tag{2.14}$$

The standard deviation has the same units as the elements of the sample.

The coefficient of variation is defined as

$$v_X = \frac{s_X}{m_X} \tag{2.15}$$

In contrast to the standard deviation and to the variance, the coefficient of variation is a dimensionless quantity. One must be careful to not confuse the variance Var(X) with the coefficient of variation v_X or the covariance Cov(X,Y) introduced in section 2.51.

The higher moments, i.e., the unbiased *skewness* d_X and the *kurtosis* e_X (a measure of the so-called flattening or conversely peakedness) are calculated as follows:

$$d_X = \frac{n}{(n-1) \cdot (n-2) \cdot s_X^3} \cdot \sum_{i=1}^{n} (x_i - m_X)^3 \tag{2.16}$$

$$e_X = \frac{1}{(n-1) \cdot (n-2) \cdot (n-3) \cdot s_X^4} \cdot \sum_{i=1}^{n} (x_i - m_X)^4 \tag{2.17}$$

2.24 Stochastic processes in time

For many quantities, not only the individual values are of interest but also their chronological sequence. One plots in this way so-called stochastic processes in time.

Meteorological data – for example, water level, water temperature, air pressure etc. – are as a rule automatically plotted on a rotating paper drum. However, it is also possible to record the data in regular time intervals, plot it graphically, and afterwards produce the graph representing the process by connecting individual points. Fig. 2/5 illustrates such a graph. For conformity reasons, here, the time axis is vertical, while values are plotted to the right.

If the characteristics of a stochastic process (mean, variance and higher moments of realisations of the process as well as the correlation between realisations at two consecutive points in time, the so-called autocorrelation) do not change with time, the process is called a *stationary stochastic process.*

However, many of today's problem areas, e.g., traffic intensity over time or melting of glaciers are *instationary*. If this case the reader is referred to the literature (see, e.g., *Ang & Tang, 1984; Castillo, 1988; Melchers, 1999).*

In the evaluation of stationary stochastic processes, two histograms are usually of interest: the histogram of the extreme values $E_i(x)$ in suitably chosen time intervals Δt and the histogram of the so-called arbitrary-point-in-time (a.p.t.) values $A_i(x)$.

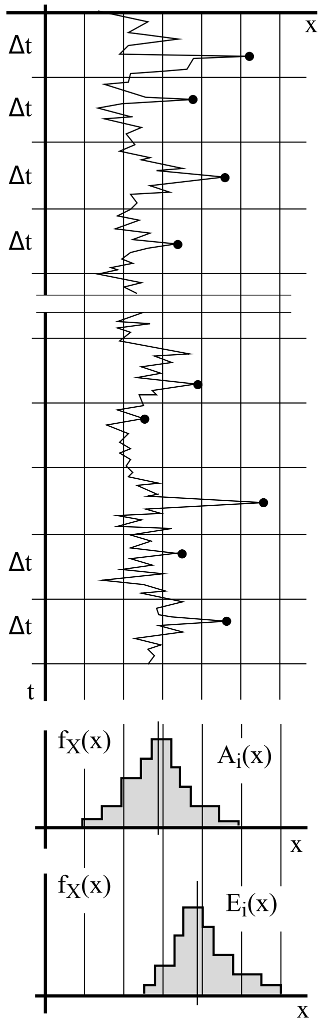


Fig. 2/5: Stochastic process

Assume the process of the water level of a river. The method of obtaining the histogram of the a.p.t. values $A_i(x)$ is then as follows: One always takes the readings of the water level (e.g., at mid-day) and derives from these the histograms according to the methods discussed previously. In this way a description of the process is obtained from which time has been eliminated. Histograms of the a.p.t. values are usually more or less symmetrical. In the case of the stochastic process of the water levels of a river or lake this is intuitively self-evident because deviations from the mean value downwards and upwards could show about the same probability. Other stochastic processes are often more complicated, e.g., the stochastic process of snow depths x, since in this case in the summer months x = 0. It can then be useful to investigate such stochastic processes under the condition x > 0.

Often, the histogram of the extreme values $E_i(x)$, i.e., of discharge during flooding, of peak temperatures, of maximum snow depths, etc., is of considerable interest. The method, again, is simple: First, the observation period is divided into equal time intervals Δt. The choice of Δt depends on the type of the stochastic process. Often a period of one year is chosen. Then, within each interval, the extreme value (maximum or minimum) is searched. From these extreme values the histogram according to the methods described previously is constructed. Again, a useful description of the process is obtained from which time is eliminated. It must be mentioned, however, that the chosen time interval Δt influences the histogram of the extreme values $E_i(x)$.

Such histograms will, in section 3.43, serve as the basis for transforming time-dependent stochastic processes into two normal time-independent random variables.

More information on stochastic processes in time or space can be found in chapter 6.3.

2.3 Distributions

Histograms reflect the properties of their underlying random sample and are not always a good representation of the entire population from which the random sample was taken. Basically, however, it is the properties of the population itself, which should be treated as a random variable. It is evident that histograms or cumulative frequencies increasingly lose their stepwise or polygon-shaped character for an increasing size of the random sample and become more or less continuous functions of the characteristic x.

2.31 Definitions

It is advantageous to replace these functions by mathematically defined *distribution functions,* because it is much easier to compute with the latter. One must be sure to distinguish between the terms *probability density function* $f_X(x)$, corresponding to a histogram, and *cumulative distribution function* $F_X(x)$, which corresponds to the cumulative frequency.

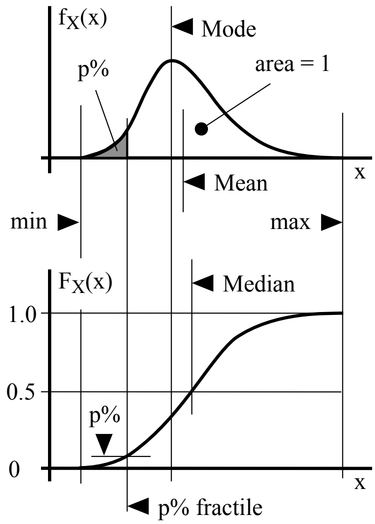


Fig. 2/6: Distribution functions

Fig. 2/6 shows above a probability density function and below the associated cumulative distribution function of a so-called continuous function. Either one completely describes a particular distribution. Often, in the literature, the abbreviations *pdf* (**p**robability **d**ensity **f**unction) and *cdf* (**c**umulative **d**istribution **f**unction) are encountered.

In fig. 2/6 two further parameters are introduced: the *median* (central value) and the *mode*. The first separates a pdf into two halves corresponding to F(median) = 0.5. The mode marks the maximum of the pdf, i.e., the most frequent value and the steepest increase of the cumulative distribution function.

In the case of symmetrical distributions the arithmetic mean, the median, and the mode coincide. The most important symmetrical distribution is the Gaussian normal distribution (see section 7.3).

Whereas the mean empirically obtained from random samples is usually designated by a Latin letter m_X, for the corresponding parameter of the continuous distribution function, the Greek letter μ_X is generally used. The same holds for the respective standard deviation s_X and σ_X.

The parameters are defined mathematically by:

$$\mu_X = E(X) = \int_{-\infty}^{+\infty} x \cdot f_X(x)\, dx \tag{2.18}$$

$$\sigma_X = v(X) = \int_{-\infty}^{+\infty} (x_i - m_X)^2 \cdot f_X(x)\, dx \tag{2.19}$$

Here, E(X) is called the expectation of X. The higher order moments, i.e., the skewness δ and the kurtosis ε, are further characteristics of a distribution function. The formulae for these parameters are derived with reference to the formulas (2.16) and (2.17). For the normal distribution $\varepsilon = 3$. Values greater than 3 indicate a distribution which is fatter in the region of the tails than the normal distribution. For symmetrical distributions $\delta = 0$. Values greater than zero indicate a left skewness distribution whose mode is less than the mean.

Often, mathematically defined distributions are abbreviated. In doing so, a capital letter represents the type, followed in parentheses by the parameters μ_X and σ_X, etc. An example is N(100;20) standing for a normal distribution with $\mu_X = 100$ and $\sigma_X = 20$.

2.32 Important continuous distribution functions

a) Symmetrical distributions

For three distribution types fig. 2/7 shows schematically the probability density functions (above) and the cumulative distribution functions (below). Further distribution types and corresponding formulas can be found in the Appendix under 7.2.

The *rectangular distribution* is the simplest of all distributions. It is bounded by the smallest value on the left and by the largest value on the right. Its use is reasonable where only "certainly greater than … certainly smaller than …" is known. The abbreviated form is R(μ;σ) or R(a;b).

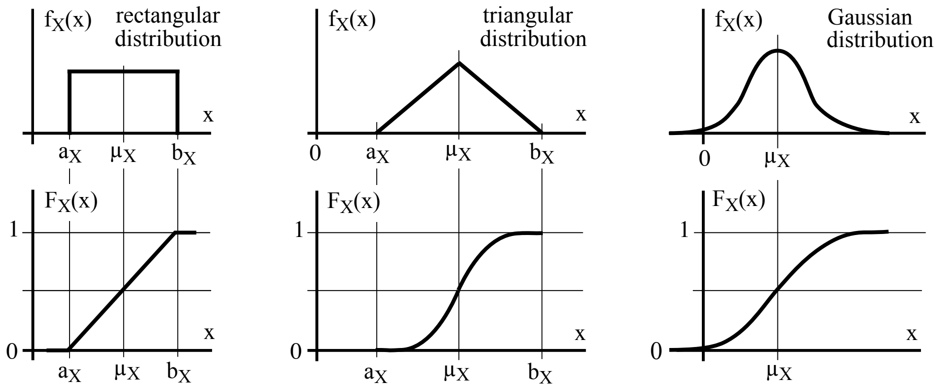


Fig. 2/7: Examples of simple distribution functions

The *triangular distribution* – T(μ;σ) or T(a;b) – does not have to be symmetrical, but it is frequently assumed. It gives mean values greater weight and would for instance be suitable, as a very good first approximation, as a replacement for the histogram of fig. 2/4.

Very many observations in nature and technology exhibit a good approximation to the *Gaussian* or *normal distribution*, N(μ;σ). The probability density function of the normal distribution – due to its shape, often called the *bell curve* – is the following:

$$f_X(x) = \frac{1}{\sigma_X \cdot \sqrt{2\pi}} \cdot \exp(-\frac{1}{2} \cdot ((x-\mu_X)/\sigma_X)^2) \qquad -\infty \le x \le +\infty \qquad (2.20)$$

Computationally, it is often advantageous to carry out a variable transformation. The standardised, so-called standard normal variable is denoted here by U. For it, $\mu_U = 0$ and $\sigma_U = 1$.

The transformation rule is written as follows:

$$u = \frac{x - \mu_X}{\sigma_X} \qquad (2.21)$$

The standard normal distribution is abbreviated as N(0;1) and is defined as follows:

$$f_U(u) = \varphi(u) = \frac{1}{\sqrt{2\pi}} \cdot \exp(-\frac{1}{2} \cdot u^2) \qquad -\infty \le u \le +\infty \qquad (2.22)$$

$$F_U(u) = \Phi(u) = \int_{-\infty}^{u} \varphi(u)du \qquad -\infty \le u \le +\infty \qquad (2.23)$$

The cumulative distribution function $F_U(u)$ according to (2.23) cannot be integrated in a closed form. The values of the standard normal distribution, however, are tabulated (see, e.g., the Appendix under 7.3). Often only a half of the symmetrical distribution is given.

b) Asymmetrical distributions

Many observations in nature, in particular of extreme values, as already mentioned in section 2.23, exhibit a skew distribution. They are characterised by the fact that the mode does not coincide with the mean value of the distribution. Of practical significance are the following skew distributions:

- Log-normal distribution
- Extreme value Type I (largest) or *Gumbel* distribution
- Extreme value Type II or *Frechet* distribution
- Extreme value Type III (smallest) or *Weibull* distribution

The extreme value distributions Types I and II are valid for maximum values (e.g., depths of snow), the Type III for minimum values (e.g., concrete strengths). The *log-normal* distribution is also often assumed for variables for which the minimum values are of interest. The corresponding formulas can be found in the Appendix under 7.2.

2.33 Discrete and mixed distribution functions

Besides the continuous distributions discussed so far there are also so-called discrete distributions, which have only discrete values, e.g., 1, 2, 3, ... A good example of such distributions is the number of trains passing over a level crossing within an hour.

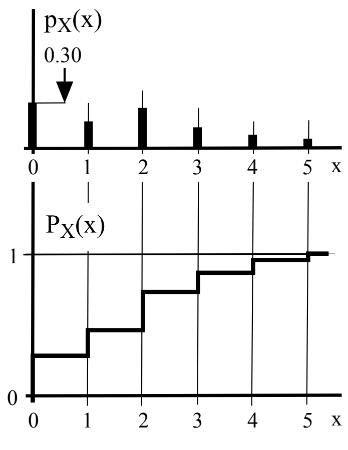


Fig. 2/8: Discrete distribution function

Fig. 2/8 shows such a discrete distribution in the form of a discrete probability (mass) function $p_X(x)$ and the corresponding cumulative distribution function. Note the difference with a density function $f_X(x)$. The figure suggests that with a probability of 0.30 no train passes the crossing while 5 trains passing per hour is a very rare event.

Obviously, in discrete distributions straight lines and jumps in the functions are typical. Needless to say, the heights of the columns in the probability density function add up to 1.

A typical example of such a discrete distribution function is the *Poisson* distribution, which is mainly used to model frequencies of occurrence of events. The Poisson distribution is defined as follows:

$$p_X(x) = \frac{h^x}{x!} \cdot e^{-h} \tag{2.24}$$

The Poisson distribution is one-parametric, as both, $\mu_X = h$ and $Var(X) = h$.

As an example: let h = 2.7 be the mean number of cars running up and queuing in front of a traffic light. What is the probability of the queue counting 5 cars? Such numbers might be of interest when designing the crossing. The answer comes as follows:

$$p_X(x=5) = \frac{2.7^5}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} \cdot e^{-2.7} = 0.08 = 8\%$$

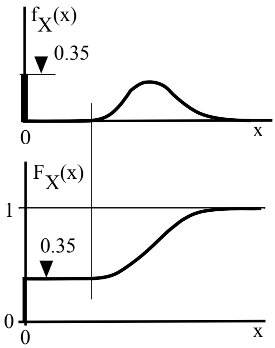


Fig. 2/9: Mixed distribution function

Also, mixed types of distribution functions are used, as shown for example in fig. 2/9. Here, at x = 0, the probability density function is a spike or delta function with an area equal to 0.35 which, as a consequence, reduces the area under the continuous part of the probability density function to 0.65, as the integral over all densities must be 1.

Such mixed distributions are sometimes used to model loads. As an example consider snow loads with values x > 0 in winter and, during the summer, a high probability that there is no snow at all.

A more detailed discussion of such distribution functions is, again, beyond the scope of consideration in this book.

2.4 Parameter estimation and extrapolation

2.41 General

The specifying of a distribution corresponding to a data set or a histogram and the estimation of the associated parameters, e.g., of μ and σ of the selected distribution, are an important area in statistics, but cannot be dealt with here in much detail.

In civil engineering the main interest is usually in small probabilities and thus in the so-called "tails" of the distributions, i.e., the shape of the probability density function far away from the mean value. A distribution which approximates a sample well in the area of the mean value may be completely unsuitable in the area of the tails. Thus, approximating the interesting tail, be it the lower tail (strengths) or the upper tail (loads), is important.

Quite often in civil engineering comparing situations, safety levels, and the like, is important. Comparisons, however, are only valid as long as basic assumptions are maintained. Therefore, often a particular type of distribution is *standardised* (e.g., log-normal or Weibull distribution for strengths, Gumbel distribution for extreme values of climatic origin, normal distribution for dimensions, imperfections of a geometrical nature etc.) even if a particular set of data suggests another distribution type.

2.42 Parameter estimation

The parameters of a distribution are, as a first approximation, determined from the parameters of the sample, i.e.

$$\mu_X \approx m_X$$

$$\sigma_X \approx s_X \tag{2.25}$$

There are often good reasons, nevertheless, to give more weight to the larger and the smaller values in a sample than to the values in the region of the mean value. Then, based on the estimated values determined from the previous formulas, one corrects according to one's judgement and thus, according to *Bayes'* way of reasoning, introduces further background information into the approximation.

In order to choose the distribution type and the parameters that best model a particular data set, the use of *probability paper* is advantageous. In this way intuitively any background information can be included and thereby the purpose of the data preparation can be considered.

2.43 Distribution fitting using probability plotting paper

Probability plotting paper – though sometimes considered old-fashioned – is still very useful for engineers. Probability papers are constructed by distorting the $F_X(x)$ axis (and if need be the x-axis itself), such that the respective distribution functions plot as a *straight line* (see fig. 2/10).

It is obvious that the extreme values on the $F_X(x)$ axis cannot be graphically presented but can be plotted "only" in the range from 10^{-n} (n circa 3 to 4) to $(1 - 10^{-n})$. From a practical point of view, however, this presentation is perfectly adequate.

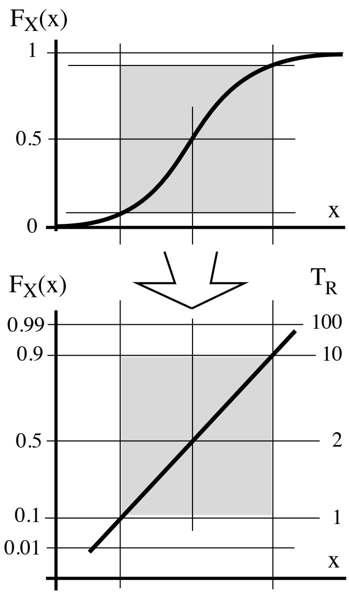


Fig. 2/10: Developing probability plotting paper

The *Normal* and the *Gumbel* probability plotting papers show a linear x axis, while the *Weibull-* and the *Log-Normal* papers show a logarithmic x-axis.

Fig. 2/11 shows, by way of example, a series of 20 measurements – e.g., the extreme values of measured wind velocities, depths of snow, etc., in a given period of time – on a normal and a log-normal probability plotting paper, respectively. How to put this data onto the probability paper is explained in the Appendix under 7.41.

One then tries by eye to draw a straight line through the series of points. The *least squares method* can be used to optimise this procedure mathematically, but usually this is not necessary. This straight line then delivers the parameters of the distribution.

The example sketched in fig. 2/11 shows that for this data set log-normal probability paper is hardly more suitable than normal paper. The series of measurements, therefore, can be described equally well by both distributions. This would not be the case, for example, for meteorological data like wind velocities, where a Gumbel probability paper would be much more suitable than a normal probability paper.

Once the distribution and the straight line have been fixed, the parameters of the distribution may be determined. This calculation is explained in detail in the Appendix under section 7.4.

A number of computer programs are of help for such data fitting (see *StatSoftware, 2016)*. It must be noted, however, that automatic data fitting might produce misleading results because either not enough emphasis is placed on the more important tails or because non-standardised distribution types are suggested and, possibly, accepted.

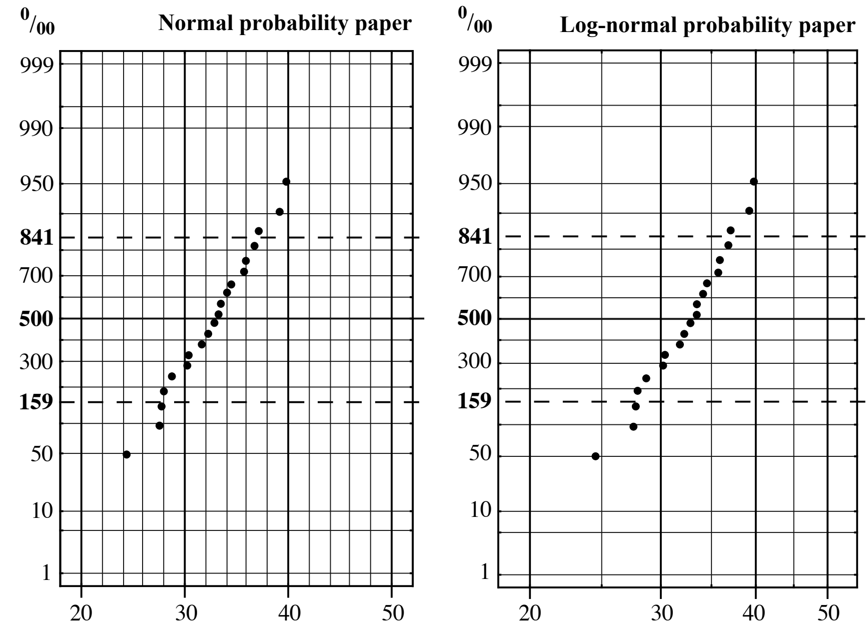


Fig: 2/11: Application of two different probability plotting papers

2.44 Extrapolations

In the course of the analysis of a set of data or of a histogram it is sometimes necessary or desirable to estimate data which lie outside the observed range. In particular, extreme values with small probabilities of occurrence are of interest. This is typically the case in, for example, hydrology and meteorology, where, despite of a lack of sufficiently long periods of observation 100 or 1000 year extreme events must be predicted by means of extrapolation.

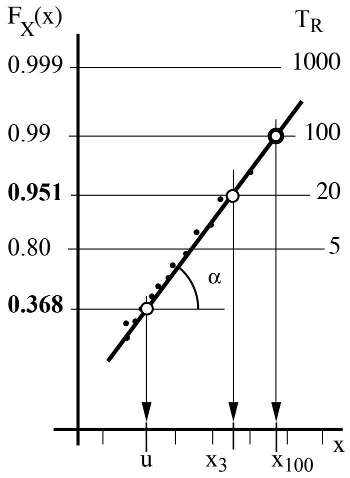


Fig. 2/12: Extrapolation

The fundamental relationship for calculating the mean return interval T_R is the following:

$$T_R = \frac{\Delta t}{1 - F_X(x)} \tag{2.26}$$

The time interval Δt is chosen according to the specific problem at hand (see section 2.24). T_R can be determined graphically or analytically.

It goes without saying that the return interval T_R has the dimension of Δt, e.g., a return interval of 100 years is based on a set of data with $\Delta t = 1$ year.

Probability paper is very suitable for extrapolations. Once the appropriate probability paper has been found, extrapolation to the left or to the right is possible. The mean return interval can also be read directly on the chosen straight line. Fig. 2/12 shows the corresponding situation for a Gumbel distribution.

The accuracy of the extrapolated values needs, of course, to be subjected to critical assessment. Extrapolations in time which are very distant from the actual series of observations (e.g., in a tenfold time period) are very questionable.

2.5 Observation in pairs and two-dimensional distributions

2.51 Problem description

Often observations are made in pairs, e.g., simultaneously both the humidity X and the air temperature Y. The question may arise whether there exists any interdependence between the quantities X and Y and, if so, to what extent.

To answer this question it is useful to plot each pair of observations (x_i; y_i) as a point in the corresponding co-ordinate system. Fig. 2/13 shows such a diagram and an isometric representation of the respective two-dimensional histogram.

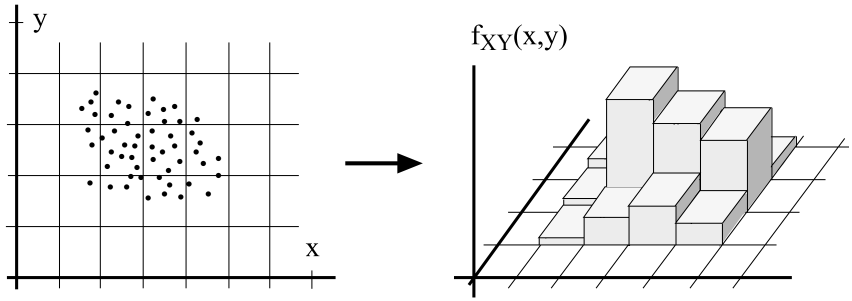


Fig. 2/13: Observation of data in pairs

This diagram gives a first, and as a rule quite clear, answer to the question of possible dependencies and shows in some circumstances the way to use the existing dependency for simplifying the analysis.

Firstly, the mean values m_X and m_Y as well as the standard deviations s_X and s_Y are separately determined as described in section 2.23. As a measure for the interdependence of both quantities the *covariance* Cov(X,Y), which is also designated $s_{X,Y}$ in short form, is defined:

$$\mathrm{Cov}(X,Y) = s_{X,Y} = \frac{1}{n-1} \cdot \sum_{i=1}^{n} (x_i - m_X) \cdot (y_i - m_Y) \tag{2.27}$$

It should be observed that $s_{X,Y}$ does *not* have the same dimension as s_X and s_Y. The standard deviations s_X and s_Y are measured in the dimension of the investigated quantity and are always positive, the covariance $s_{X,Y}$, which appears as the square of the dimension of the quantities, can take on both positive and negative values.

2.52 Correlation

The dimensionsless correlation coefficient $r_{X,Y}$ (or simply r) is calculated as follows:

$$r_{X,Y} = r = \frac{s_{X,Y}}{s_X \cdot s_Y} \qquad -1 \leq r \leq +1 \tag{2.28}$$

If r = 1, then both quantities are fully correlated. Thus X can be completely described by Y (and viceversa). With r = –1 the slope is negative and complete negative correlation exists. If r ≈ 0, the quantities – at least when only looking at the correlation coefficient – are uncorrelated, i.e., independent of one another. If some less stringent dependencies between variables exist, these show up in diagrams as partially correlated. Again, computers may be of help (see *StatSoftware, 2016)*.

Fig. 2/14 draws attention – in the bottom centre – to the fact that the correlation coefficient only recognises linear correlations. It is quite possible that correlations of a higher order are present, which can only be detected by eye (or by a more complex analysis).

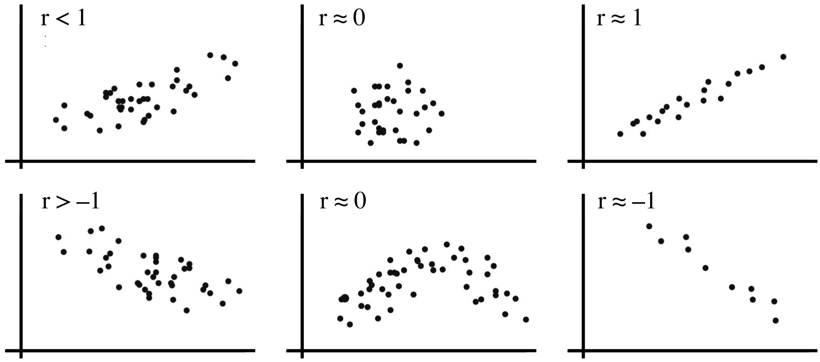


Fig. 2/14: Correlation of data in pairs. Have a keen look to the lower middle diagram

2.53 Regression

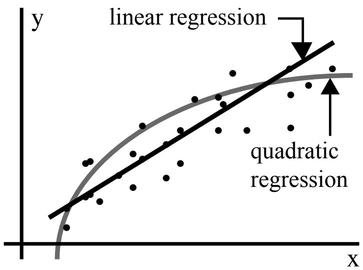


Fig. 2/15: Often, higher regression is better

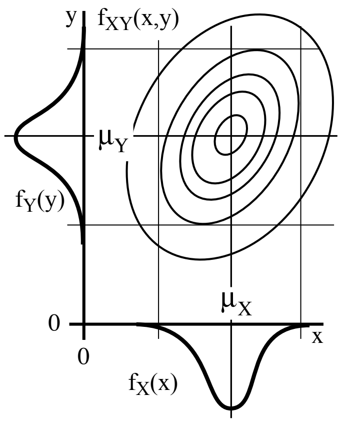
If a (linear) correlation between two quantities exists it is often of interest to express one variable by another. Fig. 2/15 shows a linear and a quadratic regression between the two variables x and y. Regression analysis normally makes use of the least squares method. Linear regression analysis is built into most pocket calculators, whereas algorithms that calculate regressions of higher order (polynomials) or exponential regressions are available in statistics programs (see *StatSoftware, 2016*.

With some skill one can, however, also draw by eye a good straight line or some other function between x and y through a cluster of points.

2.54 Bivariate distribution function

As in the transition from histograms to continuous distributions in the case of one variable, here also bivariate (two-dimensional) distributions can be defined. Both variables are considered in isolation and the parameters μ_X, μ_Y, σ_X and σ_Y are calculated according to the rules of section 2.4. The probability density functions $f_X(x)$ and $f_Y(y)$ are called *marginal densities* as they may be represented on the margin as probability density functions.

The graph of the joint probability density $f_{X,Y}(x,y)$ shows, as does the two-dimensional histogram, a "hump", represented here by contour lines as in a topographic map. From the shape and the direction of these contour lines correlation between X and Y can be detected. Fig. 2/16 shows with its sloping contour lines that the variables are positively correlated by some amount.



The covariance $\sigma_{X,Y}$ and the correlation coefficient $\rho_{X,Y} = \rho$ are calculated as follows:

$$\sigma_{X,Y} = \iint (x-\mu_X)\cdot(y-\mu_Y)\cdot f_{X,Y}(x,y)\cdot dx\cdot dy \qquad (2.29)$$

$$\rho_{X,Y} = \rho = \sigma_{X,Y}/(\sigma_X \cdot \sigma_X) \qquad -1 \le \rho_{X,Y} \le +1 \qquad (2.30)$$

For uncorrelated variables:

$$f_{X,Y}(x, y) = f_X(x) \cdot f_Y(y) \qquad (2.31)$$

Fig. 2/16: Bivariate distribution function as a hill shown by contour lines

2.6 Functions of random variables

The theory presented above can be extended to an arbitrary number of random variables. Graphical presentation is, of course, impossible in hyperspace, but the analysis, on the other hand, is possible. In the following the most important formulas are given for stochastically *independent* variables.

2.61 Computational rules

For the sum of two variables X_i:

$$Z = a + b \cdot X + c \cdot Y \qquad (2.32)$$

$$\mu_Z = a + b \cdot \mu_X + c \cdot \mu_Y \qquad (2.33)$$

$$\sigma_Z^2 = b^2 \cdot \sigma_X^2 + c^2 \cdot \sigma_Y^2 \qquad (2.34)$$

For the product of two variables:

$$Z = a \cdot X \cdot Y \qquad (2.35)$$

$$\mu_Z = a \cdot \mu_X \cdot \mu_Y \qquad (2.36)$$

$$\sigma_Z^2 = a^2 \cdot (\mu_X^2 \cdot \sigma_Y^2 + \mu_Y^2 \cdot \sigma_X^2 + \sigma_X^2 \cdot \sigma_Y^2) \qquad (2.37)$$

$$v_Z^2 = v_X^2 + v_Y^2 + v_X^2 \cdot v_Y^2 \qquad (2.38)$$

For arbitrary functions of several variables X_i of the form $Y = G\ (X_1, X_2 \dots X_n)$ no closed-form solution can be found. But, as a rule, the approximation of the function in the region of a given point x_i^* in hyperspace suffice.

To calculate these values the function – at point x_i^* – is developed as a Taylor series, whereby only the first term is taken and the mixed terms are neglected. The following simple formulas for mean value and variance – written here for uncorrelated variables – apply:

$$\mu_Y \approx G(x_i^*) + \sum_{i=1}^{n} (\mu_{X_i} - x_i^*) \cdot \left. \frac{\partial G}{\partial X_i} \right|_* \qquad (2.39)$$

$$\sigma_Y{}^2 \approx \sum_{i=1}^{n} \left(\frac{\partial G}{\partial X_i}\bigg|_*\right)^2 \cdot (\sigma_{X_i})^2 \tag{2.40}$$

The $|_*$ indicates that the value of the differential should be taken at the place of interest x_i^*. Later in this book (in chapter 4) this will be called the design point. The values x_i^* can, of course, also be taken at the mean values μ_{Xi} of the individual variables, resulting in the so-called *Taylor expansion* at the means. For this

$$\mu_Y \approx G(\mu_{Xi}) \tag{2.41}$$

while $\sigma_Y{}^2$ still follows from eqn. (2.40).

For example, given the function:

$$Y = a + b \cdot X_1 + c \cdot X_2 \cdot X_3{}^2$$

the partial differentials are:

$$\frac{\partial Y}{\partial X_1} = b \qquad \frac{\partial Y}{\partial X_2} = c \cdot X_3{}^2 \qquad \frac{\partial Y}{\partial X_3} = 2\,c \cdot X_2 \cdot X_3$$

From these partial differentials, restricted to uncorrelated variables, at the point with the coordinates $x_i^* = \mu_{Xi}$, the mean and the variance are obtained:

$$\mu_Y = a + b \cdot \mu_{X1} + c \cdot \mu_{X2} \cdot \mu_{X3}{}^2$$

$$\sigma_Y{}^2 \approx b^2 \cdot \sigma_{X1}{}^2 + (c \cdot x_3{}^{*2})^2 \cdot \sigma_{X2}{}^2 + (2c \cdot x_2{}^* \cdot x_3{}^*)^2 \cdot \sigma_{X3}{}^2$$

2.62 The central limit theorem

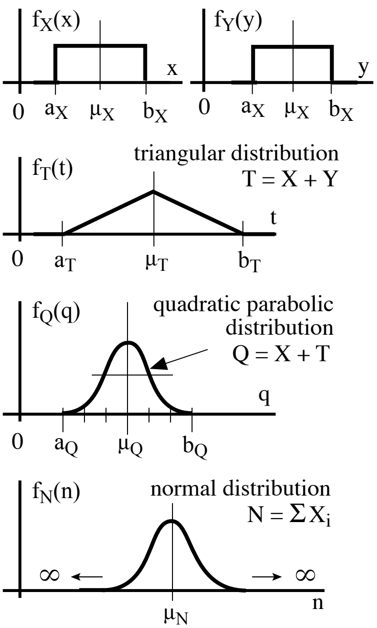


Fig. 2/17: The central limit theorem

It can be shown that sums of symmetrical distributions remain symmetrical, while products of symmetrical distributions exhibit a skew shape. The central limit theorem provides useful information on the shape of the probability densities for sums and products of independent variables. The following theorems apply provided that none of the variables dominates:

- The distribution of the *sum* of n arbitrary random variables X_i approaches the *normal* distribution, independent of the distribution types of the variables, with increasing n.
- The distribution of the *product* of n arbitrary random variables X_i approaches the *log-normal* distribution, independent of the distribution types of the variables, with increasing n.

It is not surprising, therefore, that in nature there are many observations that seem to obey either a normal or log-normal distribution, depending on whether they result quite naturally from the sum or the product of the quantities influencing their behaviour.

The first theorem may be illustrated as follows: If summed, two rectangular distributions of the same width (fig. 2/17) result in a triangular distribution. Now if another rectangular distribution is added, a bell-shaped curve results, whose shape is made up piecewise of parabolic sections. The shape of this bounded distribution is already quite close to the shape of the normal distribution.

By means of suitable software, the validity of the two theorems may be seen using the Monte-Carlo method treated in section 4.2.

2.63 Further parameters of functions of variables

The literature reveals several different methods for the approximate determination of the moments of a function of variables. A quite accurate numerical method is the Point-Estimate Method (see e.g., *Evans, 1972; Zhou & Nowak, 1988; Li, 1992)*. Here, function values are calculated for defined realisations of the variables and summed up using selective weighting. The detailed presentation of these integration methods and the interpretation of the results are beyond the scope of this book.

2.7 Fuzzy Information

Not only numbers but also words, i.e., verbal expressions about the quality of some situation carry information. When writing here about information processing the authors cannot escape from shortly touching on the term and concept of *Fuzzy Information*.

The word *fuzzy* stands for diffuse, cloudy, vague, etc, information used in our daily talk and conversation with others. *Fuzzy* are verbal qualifying statements about a state, a measure, a situation, such as *"too big"*, or *it's "hot" today*, or *it's "quite far" away*, or *here we are "safe"*. Also between engineers fuzzy talk is normal and often not even realised as such. On the other hand it is understandable that a person keen to express himself clearly and reliably fuzzy expressions are worthless or even intolerable. Two examples may clarify the basic idea and problem behind what is discussed here:

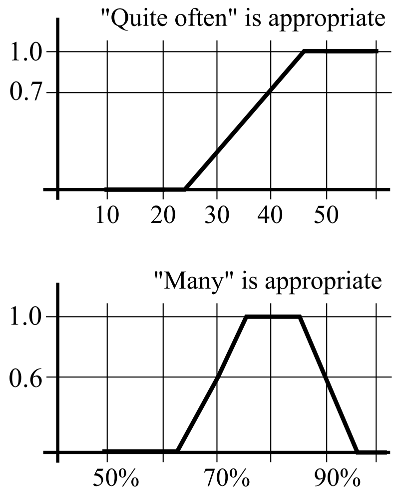


Fig. 2/18: Examples of fuzzy information

Upon a question about the yearly frequency of a specific event the reply was: *Oh, this happens "quite often"*. This statements transports his/her, opinion about the frequency and it's clear to him or her that this verbal statement is not very precise. All those confronted with this imprecise statement about the frequency of such an event translate it into their own set of opinions of what might be *"quite often"*. Some may translate the statement into 20 times, others may think that more than 40 times might be *"quite often"* and 20 times just "*not rare"*.

In this sense *"quite often"* and *"not rare"* is *fuzzy,* like many other qualifying expressions are used, not the least also in technical talk, and often quite successfully, such as *good, bad, important, hazardous, appropriate,* etc.

To stay with the above example: Maybe one could conclude within the group of those discussing the case that *"quite often"* in the above context may be defined by the so-called membership function shown on top of fig. 2/18. A frequency of 40 times per year would be associated to

0.7 to the sub-group *"quite often"* and to 0.3 to *"very often"* while 20 times would be associated with the sub-group "not rare".

In the same sense, *"many"* may be defined within the scale from *"none"* to *"all"* by the membership function shown on the bottom of fig. 2/18. Verbally explained this is: *"Many"* is by 70% less than 0.6. *"Many"* would be quantities between 75% and 85%, while 90% would be classified by the next category, e.g., *"almost all"*.

What was shown above for *"Many"* could easily be used for the description of possible consequences of a hazardous event, classified by *"very small – small – average – large – very large"*. Maybe choosing a logarithmic scale would be necessary to cope with the different grades of consequences.

As can be seen, first of all a quality scale must be chosen. Then, the membership functions must be defined. Done this, the door is open to exercise what is called *Fuzzy Set Theory.* This theory found quite a number of friends. Interested readers may consult the literature like, e.g., *Zadeh, 1965; Blockley, 1975; Zimmermann, 1991*.

However, the definite view of the authors of this book is that within the scope of a scientific approach to engineering the use of the Fuzzy Set Theory should be discouraged. To their belief there is simply no problem that is "solved" by Fuzzy Sets and that cannot be better handled by probability tools. If, e.g., somebody states that an event might happen *"quite often"* per year, the better way is the endeavour to find out what the probability density function is behind that person's opinion about the frequency.

The big advantage of using probabilistic methods is that, in principle and given sufficient observations, all kinds of such statements can be analysed and brought into a successful probabilistic context.

3. Basic Variables and Modelling

3.1 Introduction

A whole range of problems in civil engineering can be described by the comparison of two stochastic quantities: one, some sort of solicitation or stress (hence called the S variable); the other, a corresponding capacity or resistance (hence called the R variable). The following examples illustrate the point:

R	S
flow capacity of a river bed	discharge of the river
flow capacity of a sewage pipe	discharge of waste water
bending resistance	existing bending moment
permissible deflection of beam	existing deflection of beam
soil cohesion and shear strength	stresses in soil due to external loads
traffic capacity of a road junction	intensity of traffic

As a rule it is expected that the quantity on the left, the R variable, is at least as big as the quantity on the right, the S variable, so that no failure occurs. In terms of the examples: the river does *not* overflow its banks, the beam does *not* fail, the slope does *not* become unstable, the traffic does *not* come to a standstill, there is *no* electrical power failure, etc. From the examples it follows that such comparisons might consider a situation, or be a matter of time.

Checking for structural safety, e.g., traditionally follows *deterministic* patterns. In principle, a defined value r_d of the resistance of a structural component is derived from a number of characteristic values. In a similar manner a defined value s_d representing the action effects is derived from a number of characteristic values of actions. In order to check for safety or failure these two single values r_d and s_d are then compared.

The *deterministic* or *semi-probabilistic* form of the safety condition reads:

$$r_d \geq s_d \tag{3.1}$$

Sometimes a conscientious engineer repeats such an analysis in order to test the sensitivity of the result to variations of the input values. This is a step in the right direction, but it is often cumbersome and does not result in a good overview.

In the *probabilistic* approach advocated here the quantities that influence the problem are introduced as variables with their distribution types and their respective parameters. All load and resistance factors are dispensed with. Their function, however, is partially accounted for by so-called model variables.

Using R and S as variables in the above sense, the normal or desired state can be formulated as follows:

$$R \geq S \tag{3.2}$$

or rearranged:

$$R - S \geq 0 \tag{3.3}$$

Failure occurs when:

$$R - S < 0 \tag{3.4}$$

In these expressions R and S are *stochastic variables* representing, in terms of structural engineering, e.g., the resistance of a section and the stress in this section due to applied loads and actions, respectively.

The following paragraphs present examples relating to the design of new or the assessment of existing structures. The methodology and the conclusions reached easily apply to other technical areas as well.

3.2 State, components, basic variables

3.21 Basis of assessment

The structural system shown in fig. 3/1 will serve as an example. As a rule such a system has to be designed or assessed with regard to its structural safety and its serviceability.

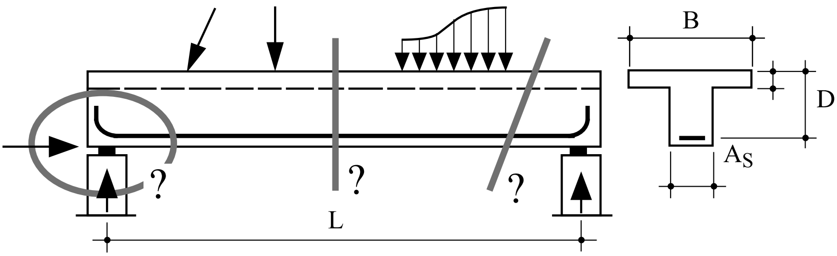


Fig. 3/1: Reinforced concrete beam and three failure possibilities

The verification of *structural safety* relates mainly to bending moments in the critical section somewhere at mid-span and the shear forces at the supports. The beam may fail in either way. Also, the anchorage of the reinforcing bars at the supports has to be checked.

A beam complies with the *serviceability* requirements if it does not show excessive deformations under normal loading, if it does not vibrate in a disturbing manner, and – e.g., in the case of reinforced concrete – if the crack widths remain within acceptable limits.

The behaviour at the centre of the beam, for example, may be assessed by comparing R and S. Of course R and S depend on a number of quantities. Fig. 3/2 illustrates some of these quantities. To be sure, the behaviour may be better and more accurately assessed on a more detailed basis. Instead of two variables, about a dozen would then have to be included in the analysis.

Naturally, all these quantities depend in turn on others. The concrete strength β_c, for example, can be further subdivided: it is a function of the quality of the gravel and cement, the water-cement ratio, the quality of pouring and compaction, the temperatures in the hardening process, etc. An investigation on this basis will then contain many variables and the analysis would become extremely and unnecessarily complex.

Somewhere, this branching-off process has to be terminated. In many cases a comparison on the basis of R and S alone is possibly too inaccurate. On the other hand, a fine division into many basic variables is more "accurate" but also more complicated and cumbersome. Choosing the optimal basis of assessment therefore is always an issue to be addressed.

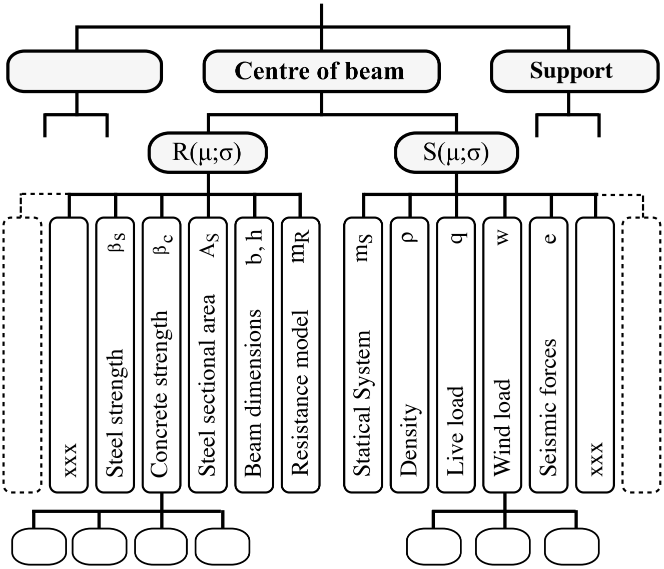


Fig. 3/2: Elements of a reliability assessment

As a rule the basis of assessment should include as few variables as possible. The variables of the finally chosen basis are called the *basic variables.*

3.22 Basic variables

There are three types of basic variables:

- *Environmental variables:* Wind, snow, ice, earthquake, temperature, etc., are stationary, time-dependent stochastic processes. They are generally not controllable by man. Hazards from fire and explosion induced by human activity also belong to this category. The fixing of design values for environmental variables is equivalent to accepting certain risks.
- *Structural variables:* Structural dimensions, structural materials, etc. are planned. They are amenable to checking and can if necessary be improved by replacing them. As a rule, structural variables do not vary at all or only very little with time and are on the whole, apart from corrosion processes, fixed quantities. The prediction of these more or less fixed quantities is nevertheless difficult. This is the reason why structural variables are considered here as *stochastic* quantities.
- *Utilisation variables:* live loads, traffic loads, crane loads etc. can be controlled by supervision. They are generally time-dependent stochastic processes. Though the service criteria agreement (see section 1.42) generally contains agreed-upon values for utilisation variables, they are still treated as variables because in practice uncertainties exist regarding the keeping of such agreements. It must be stated, however, that planned *changes* of utilisation require a new investigation.

An alternate subdivision into R variables and S variables is sometimes useful:

- *R variables* normally occur on the resistance side: dimensions, strengths, storage volumes, friction coefficients, cohesion. Usually for R variables, values *below* the mean value are dangerous.
- *S variables* are normally encountered on the loading side: loads, forces, inflows, amounts of precipitation. Usually for S variables, values *above* the mean value are dangerous.

A strict separation of R and S variables is not always possible. In the case of a retaining wall, for example, the soil properties are involved both on the active and the passive sides. However, within the framework of reliability theory, a separation is also not necessary.

From the set of basic variables a subset of variables is selected that are actually treated as variable quantities. They are introduced with at least two parameters (e.g., mean μ and standard deviation σ). All other basic variables are deterministic – i.e., will be introduced as fixed values (nominal value, planned value, mean value, ...) – in other words with the standard deviation $\sigma = 0$.

Basic variables are in many cases statistically independent. They may, however, also be statistically dependent or correlated. The bending resistances in different cross-sections of a beam, e.g., are very strongly correlated. A smaller correlation coefficient will exist between the properties of different steel beams, and even a smaller one between those of different manufacturers. An inverse relationship of size and strength applies for steel reinforcing bars.

In general, statistical independence of the variables will be assumed in this book.

3.3 Resistance of structural elements

The abbreviation R denotes the resistance of structural elements in a given cross-section of a structure, or it can stand generally for some other capacity of the system under consideration (see section 3.1).

3.31 Resistance model

The reasoning will be illustrated for the structural resistance R. The model for R has, as a rule, the following typical form:

$$R = M \cdot F \cdot D \tag{3.5}$$

in which:

M = model uncertainty variable

F = material properties (strength, elastic modulus, …)

D = dimensions and the derived quantities

The parameters of R are determined using the computational rules of section 2.61:

$$\mu_R = \mu_M \cdot \mu_F \cdot \mu_D \tag{3.6}$$

$$\sigma_R = \sqrt{\sigma_M{}^2 + \sigma_F{}^2 + \sigma_D{}^2} \tag{3.7}$$

As already mentioned earlier resistance tends to a log-normal distribution, since it appears as a product of variables. Besides, negative resistances are hardly possible.

Detailed information on R variables may be found in various publications of the JCSS (see *JCSS 1981 and later)* and in *Spaethe, 1992*, in which the results of extensive data bases are summarised. For steel structures, *Petersen, 1977,* is useful, while for concrete structures, *Mirza & MacGregor., 1979,* is helpful. Of particular interest are the probabilistic models and the parameters of the corresponding design variables produced for the Dutch building codes (see *Vrouwenvelder & Siemes, 1987)*. The most recent information is supposed to be present in the JCSS Probabilistic Model Code (see the *JCSS Website)*.

3.32 Model uncertainties

Since in developing a resistance model certain influences are either consciously or unconsciously neglected, deviations between analysis and tests are to be expected. This fact is considered by introducing a model variable M that may be determined from tests. The test results r_{Exp} are divided by the corresponding results r_{Mod} obtained using the resistance model:

$$m = \frac{r_{Exp}}{r_{Mod}} \tag{3.8}$$

From a number of tests the mean value m_M and the standard deviation s_M and a histogram for M is obtained. These experimental results are then replaced by a suitable distribution.

For good models $\mu_M \approx 1$ is obtained. Since conservative models are frequently used, it follows that often $\mu_M > 1$. The value of σ_M differs greatly depending on the quality of the model. For good models (e.g., for the bending resistance of steel and reinforced concrete sections) the coefficient of variation is just a few percent, whereas for poor models (e.g., the shear and the punching resistances of reinforced concrete structures) values in the region of 10% to 20% may be typical.

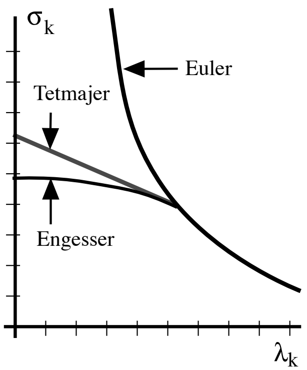


Fig. 3.4: Model uncertainty

Furthermore, the parameters of model variables are sometimes variable in the design range. Consider, e.g., the buckling stress σ_k of a compression member and plot this against the slenderness λ_k of the member (see fig. 3/3). For high slenderness ratios the expected deviations from the Euler buckling curve are small.

For small slenderness ratios, on the other hand, many factors not accounted for in Euler's theory become more noticeable (e.g., residual stresses, shape of cross-section, undesirable eccentricities, imperfections). The model uncertainty for lower slenderness is, of course, strongly influenced by the choice of the resistance model (e.g., the one by *Tetmajer, Engesser-Karman, Engesser-Shanley,* or using the *European Buckling Curves)*. Improved models contain as a rule model variables with a smaller standard deviation and mean values close to 1.

3.33 Material properties

The values for strengths F and other material properties are obtained mostly from tension and compression tests. Usually the results cannot be used directly because of the following problems:

- Conditions in a laboratory test are often quite different from those in the structure.
- The scatter in the material properties of the structure is usually greater than the scatter in results from laboratory tests.
- Material properties may vary in time.

The simplest method of dealing with these influences is to use the following relationship:

$$F = P \cdot T \tag{3.9}$$

P denotes the properties variable, T a so-called transfer variable. The properties variable P represents what is effectively measured. In testing laboratories, histograms for P as well as for its parameters m_P and s_P are determined and an appropriate distribution type is chosen.

The dimensionless transfer variable T takes into account the ratios of the properties of a structural component and the respective quantity measured in the test. In the case of concrete strengths, the difference between the prism and cube strengths as well as the ratio of the strength in the structure and that of concrete cubes stored according to the relevant building code for a period of 28 days enters into the transfer variable. As a rule, transfer variables have a mean value smaller than unity and a coefficient of variation of some 10% to 15%.

Thus the parameters for F amount to:

$$\mu_F = \mu_P \cdot \mu_T \text{ and} \tag{3.10}$$

$$\sigma_F = \sqrt{v_P{}^2 + v_T{}^2} \tag{3.11}$$

The value μ_F should *not* be confused with μ_P. Neither of these values corresponds with the computational value f_{Nom} laid down in codes, for instance. Generally, the latter contains deterministic corrections on the conservative side.

3.34 Geometrical properties

Geometrical properties D may be measured directly. The dimensions may be checked and compared with the corresponding tolerances. The mean values are usually close to the nominal values. Occasionally systematic influences arise. For example, the pressure on the formwork during concreting can cause it to deform, so that the actual dimensions exceed the planned dimensions. The standard deviations σ_D are of the order of magnitude of the dimensional tolerances and are thus largely independent of the absolute dimensions of the cross-sections. Thus the coefficient of variation is bigger for smaller dimensions.

3.4 Action effects in structural elements

3.41 Actions

When talking about actions the following terms, e.g., for wind action, have to be distinguished:

- Influence: v_{wind} [m/s]
- Action: w [kN/m^2]
- Action effect: M, V, N … due to wind

By way of example, a number of actions normally taken into account in the design of structures is shown in fig. 3/4 plotted as concurrent stochastic processes. Again, action values are plotted along the horizontal axis while the time axis is vertical.

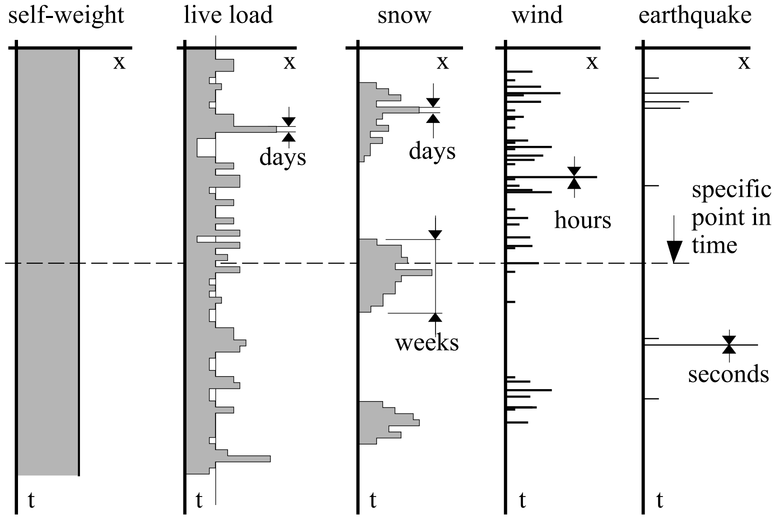


Fig. 3/4: Various actions on a vertical time scale

The processes illustrated in fig. 3/4 are briefly discussed below:

- The *self-weight* may to a good approximation be regarded as constant over time.
- Each structure supports in addition to the self-weight *quasi permanent* live loads. These are superimposed by *short term* live loads. In buildings the latter have a duration in the order of hours or days. On a bridge, e.g., high values of loading occur at known times (rush hours) and reach peak values during traffic jams and accidents. These maximum values last from a few minutes to several hours.
- In lower lying areas of many countries there is *snow* for only a short period in the winter months. Maximum values are recorded over periods of days, average values over periods of weeks and months. In mountainous regions the snow may lie on the ground for longer periods and, for snow-prone countries, may be considered to exhibit the character of permanent influence. In some countries snow may be only exceptional or completely non-existent.
- *Wind* only occurs for short periods, say, several minutes to a few hours. Strong gusts are seldom. Maximum gusts last for only a few seconds to minutes.
- Finally, *earthquakes* occur very rarely. Their period of strong motion is in the region of several seconds. The intensity is highly variable.

3.42 Modelling of actions

Actions are as a rule stochastic processes in time. As shown in section 2.24, such processes can be represented by two stochastic variables. From the histograms derived from the observed data, two different kinds of variables are defined, e.g., leading and accompanying actions (see fig. 3/5).

The leading action is determined essentially by analysing the relative stochastic process with respect to its extreme values e. Usually these exhibit an extreme value distribution $E_i(e)$, which is defined by its type, e.g., a Gumbel distribution together with the respective parameters.

Accompanying actions are essentially derived from the arbitrary-point-in-time (a.p.t.) values of the stochastic process and exhibit a more or less symmetrical distribution $A_i(x)$ with respect to the mean and are normally modelled by normal or log-normal distributions.

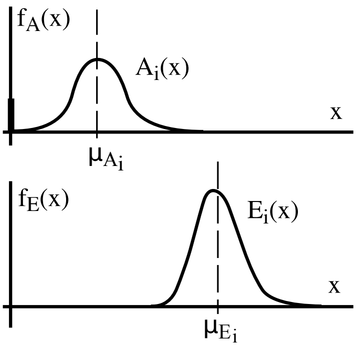


Fig. 3/5: Modelling action

There might be "off-times", e.g., for snow loads, wind, or earthquake, where the process shows zero action. This is shown in fig. 3/6 by the discrete density at x = 0. It is conservative to ignore off-times, however, and to derive the $A_i(x)$ distribution from the times during which the process exhibits action.

From a stochastic process in time, a sample of values x_i may also be derived, for instance, in reading the action at specified time intervals. These values can be sorted by increasing values. From this sorting a continuous curve as shown in fig. 3/6 and valid for the observation time T can be drawn. This is the so-called Load-Duration-Curve. It shows how long an action exceeds a certain level during the life-time T. Since there may be within T some intervals where the action is zero, a respective diagram starts at t_0. It is obvious that at the right of the figure the larger values correspond essentially to the extreme values of the process. Therefore, the shape of the diagram in this region depends on T. The intermediate part of the diagram represents the a.p.t. values and is not influenced by T.

Dividing the time axis by the total observation time T, the probability distribution function $F_X(x)$ of the process can be derived (see again fig. 3/6). It is within the nature of actions that they often cannot be described by continuous functions. In this case the respective pdf can be approximated by a histogram. Both the continuous as well as the discrete form lend themselves to Monte Carlo simulation techniques (see, for instance, *Marek et al., 1995)*.

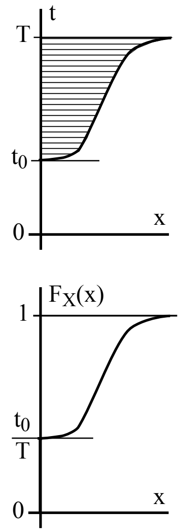


Figure 3/6

3.43 Combination of action effects

A horizontal line in fig. 3/4 denotes a particular *point in time* in the life of a structure. On this line the individual, simultaneously occurring actions can be read. From these, the corresponding action effects in the parts of the structure of interest may be determined and summed up. The result of this summation is, e.g., a sectional force. Moving the line arbitrarily up and down, somewhat randomly, the highest value of the summation and thus the maximum value of stress in the period of time considered may be found. This method, however, is obviously cumbersome and therefore unacceptable as a procedure.

A number of methods have been derived to solve this problem. Here, only the application of *Turkstra's* rule and the *Ferry Borges-Castanheta* action model (FBC action model) will be discussed.

a) Turkstra's Rule

The action processes may, of course, be searched for some defined maximum in a systematic manner. A procedure proposed by *Turkstra*, *1972,* has found its way into practice and is known today as "Turkstra's rule". One of the action processes is chosen and denoted *leading action*. At the point in time where this process reaches its maximum, the values of all other so-called *accompanying actions* are read. The leading action, together with its accompanying actions, define a so-called *hazard scenario* (see also section 1.34).

Each action, in turn, is considered as a leading action. Thus there are as many hazard scenarios as there are actions that occur simultaneously. Obviously, the most unfavourable hazard scenario for a particular quantity is the critical one.

Of course, the action effects have to be expressed by a common measure, be it the sectional forces or any kind of generalised stresses. Thus, the action effect S is calculated as a function of the different actions. Written in a general way it has the following structure:

$$S = \left[S\left(L_i + \sum_{\substack{j=1\\ j\neq i}}^{n} A_j\right)\right]_{\max} \quad \text{seek max by choosing } i \text{ from 1 to n} \tag{3.12}$$

in which

$$L_i = M \cdot M_{Ei} \cdot E_i \qquad \text{leading action} \tag{3.13}$$

$$A_j = M_{Aj} \cdot A_j \qquad \text{accompanying action} \tag{3.14}$$

M, M_{Ei} and M_{Aj} are model variables, which will be discussed in section 3.44.

Turkstra's rule leads to results which, from the theoretical point of view, lie on the unsafe side, because it is quite possible that the most unfavourable situation occurs at a time that is not characterised by the maximum of any of the action processes. It is obvious, however, that with this rule the most unfavourable situation is closely approximated.

Turkstra's rule is the underlying concept of the terms *hazard scenario*, and *leading* and *accompanying actions*. The concept can, however, be much more generally applied. This becomes evident if the term *action* is replaced by *hazard* and thus freed from the narrow connection with load combination. The *Hazard Scenario concept* (see *Schneider, J., 1985)* is quite generally applicable and especially useful in hazard identification and the definition of *design situations* in structural design.

b) The Ferry Borges-Castanheta model

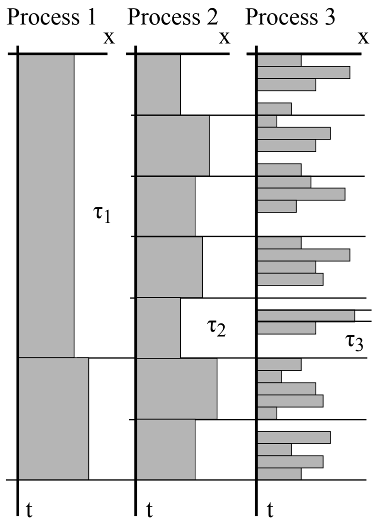


Fig. 3/7: Ferry Borges-Castanheta model of actions

Often, instead of Turkstra's rule, the load combination rule based on the Ferry Borges-Castanheta action model (see *Ferry-Borges & Castanheta, 1971)* is used. This model represents each individual stochastic process in the form of a series of rectangular pulses (see fig. 3/7). The value of such a pulse represents the intensity of the load. This is a random value from the a.p.t. distribution of the action. The duration of the pulse τ_i remains constant within the series. This time interval τ_i is chosen such that the pulses can be considered as independent repetitions of the respective actions.

Time intervals of different processes are chosen such that the longer interval is an integer multiple of the next shorter one, valid for all processes involved. This is a prerequisite for easy calculation of the maximum value distribution of the combination of two processes because pulses of the shorter-step process are repeated exactly n-times within the pulse duration of the longer-step one.

Typical intervals chosen are, e.g., 12 hours or a day for wind, a week or a month for snow (depending on the country and the region within the country), seven years for imposed long-duration loads, etc., whereas the life-time T of the structure is often assumed to be, e.g., 50 years, which is about seven times seven years.

Consider the case where three action processes X_1, X_2 and X_3 are acting on a structure. The Ferry Borges-Castanheta action model then considers as a new variable the maximum of X_3 within an interval τ_2 together with X_2. This variable, in turn, is searched for its maximum during an interval τ_1 and added to X_1. From this, finally, the maximum during the life-time T is considered as the variable representing all three processes together.

The variable Y representing the maximum combined action of the three processes thus may be written as follows:

$$Y = \max_T\left\{X_1 + \max_{\tau 1}\left[X_2 + \max_{\tau 2}\left(X_3\right)\right]\right\} \tag{3.15}$$

Herein, the X_i represent the a.p.t. distributions. The terms $\max(X_i)$ represent the maximum values of the random variables X_i within the periods τ_i or T, respectively. Or, in other words, the index of max represents the process from which the maximum should be searched.

3.44 Model uncertainties

The model variable M introduced in section 3.43 a) takes into account the uncertainties introduced into the analysis by simplifications, e.g., of the statical system, of the load pattern or shape, and of the influences of stiffness and cracking of structural parts.

Regarding *serviceability,* considerable uncertainties exist. Depending on the problem, the following could be the parameters of the model variable:

$m_M \approx 1.0$
$v_M \approx 0.05$ up to about 0.3

Regarding *structural safety,* the consideration of an equilibrium state is of paramount importance. Whether the chosen equilibrium state is close to or far from reality as reflected by elasticity theory is not as important because uncertainties in the action effects in different sections tend to cancel each other out. Thus, $\mu_M = 1$ and $v_M = 0$ may be safely adopted.

For simplicity's sake, the model variable M which accounts for model uncertainties in the determination of S, is considered together with the leading effect.

The model variables M_{Ei} and M_{Aj} of the *leading* and *accompanying actions* respectively are action-specific and have to be defined together with the corresponding actions. They account for the differences between the real actions (whatever they are) and their respective model.

Model variables are typical *Bayes* variables. They may be derived from tests as shown in section 3.32. But often there is no way other than to estimate the parameters in a quite subjective way.

3.45 Some comments on actions

It is not possible here to discuss in detail the results of research on actions. Brief comments, which should be regarded as a basis for further thinking, have to suffice. Reference may be made to the studies of Commission W 81 of the *CIB* (see *CIB, 1989* and *1991)* as well as to the JCSS

Probabilistic Model Code (see the *JCSS Website).* It is also mentioned that often useful statistical background may be found in studies related to (updating of) regional or national loading codes.

a) Permanent loads

Self-weight is essentially constant over the life span of the structure. There is a small tendency to higher loads because of tolerances, deformation of the formwork, etc. Thus, for example, as a mean value a factor 1.05 of the nominal value is often assumed, along with a coefficient of variation of about 5%. The assumption of a normal distribution is adequate. The dead loads of a structure are only exceptionally a leading action.

Other *permanent loads* are also practically constant over long periods (decades). The standard deviations, however, are often greater than for dead loads. Considerable changes are possible for various reasons: application of additional surface layers, thicker layers, heavier installations, or the removal of installed parts. Such changes cause relatively *large changes of the mean values*.

In the case of *actions resulting from fill material,* the *method of filling* is important. In some circumstances heavy construction machines are used. In addition the danger exists that large amounts of earth have to be temporarily placed in a heap and then distributed in the prescribed thickness. Therefore, regulations for placing the fill material have to be fixed in the control plan and enforced during execution. For the coefficients of variation – depending on the amount of control – a value of 20% to 30% is quite realistic. As a type of distribution, the log-normal distribution is often assumed.

b) Earth and water pressure

Earth pressure is basically a function of the density of the fill material and of the angle of internal friction. The mean value of unit weight has to be taken from the corresponding handbooks. Coefficients of variation of about 7% and more are to be expected. The mean value of the angle of internal friction to be adopted is the most probable value; the coefficient of variation is about 10%. It should be noted that geotechnical engineers may already have safety considerations built into their data. Thus, the nature of the information in the geotechnical report in each individual case should be discussed.

Hydrostatic pressures clearly depend on the position of the water table. Thus the variations of the water table have to be monitored and converted to suitable distribution functions. Often the maximum height of the water table is physically limited. This should not be misunderstood to imply that no further safety considerations are necessary. On the contrary: the necessary safety reserve is simply shifted to the resistance side. This corresponds to conventional design rules, that – even for actions with a physical upper limit – require the application of load factors larger than 1 to the upper load limit.

c) Live loads

The permanent part of *live loads* in buildings (i.e., loads resulting from installations and normal operation) is relatively small and is normally only an accompanying action (see fig. 3/5). Short term peaks are the result of exceptional gatherings of people, crowding together of installed objects (renovation, furniture removals), or the failure of installations (burst water pipes, etc.). The variation applies over all the floors of the building (see *Sentler, 1976).* In order to describe loads as accompanying actions the moments μ_A and σ_A of the a.p.t. distribution (see section 2.23) may be applied.

Accompanying actions arising from live loads are often assumed to exhibit a log-normal distribution.

The leading action due to live loads in buildings is often given by the moments μ_E and σ_E for the peak value for a crowd of people. With increasing mean μ_E, the standard deviation σ_E of the load decreases as a result of decreasing possibilities for crowd movement.

As a rule these values have to be introduced into the analysis as extreme value distributions of some suitable type.

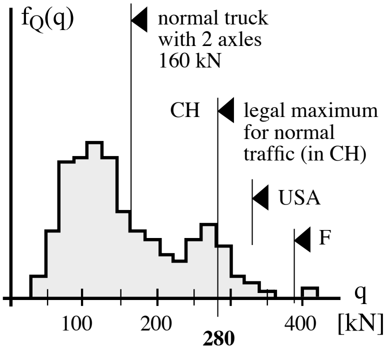


Fig. 3/8: Traffic loads on bridges

A study of *traffic loads on bridges* (*Jacquemoud, 1981)* gave the interesting distribution of fig. 3/8. It derives from 2'340 measurements and exhibits two peaks, which are easily explained: The left peak is produced by the average traffic load, while the second peak is caused by trucks loaded up to the present legal limit in Switzerland, a total weight of 280 kN. A small third peak results from a higher weight category for trucks (in the EU) of 400 kN, which exceeds the present Swiss limit.

The left peak is characteristic of an *accompanying* action, the right one, with full exploitation of the permissible loading, represents the *leading* action. Finally, bridge loading is the result of a number of influences, among others the load distribution on the axles; the wheel base; the lateral position on the highway; dynamic effects; and, especially, the conditions during a traffic jam.

d) Snow loads

Snow is one of the most important types of load in many countries. Every year many roofs collapse under snow loading. Constructions with smaller self-weight (and thus *de facto* small reserves) in areas with particularly heavy snowfall are especially endangered.

Snow loads depend on climatic conditions (temperature, rainfall, sunshine), snow drift, shape of roof, heating etc. Strong regional and geographical influences have to be considered. An excellent discussion of the problems involved and a detailed list of references is given in *CIB, 1991. Ghiocel & Lungu, 1975* and *Sanpaolesi, 1999,* may also be of further use.

Snow load models normally consider two steps. At first, the snow load on flat ground is determined. Here, the depth of snow and the density of snow are random variables. The density differences between light powder snow and wet snow are immense. Using *water equivalents* of the snow would be more reliable. Unfortunately, measuring water values was only a matter since two or three decades. Thus, the series of observations do not yield sufficient information.

The second step considers snow loading on roofs. This, for various reasons, often differs considerably from that on flat ground. One must consider influences of exposure and various melting processes resulting from heating of roofs and insulation capacity. In addition the various forms of snow accumulation in unfavourable areas of the roof have to be taken into consideration.

Series of depth measurements of snow should be evaluated using log-normal paper and those of snow water values using Gumbel paper (see section 2.43).

e) Wind forces

Everywhere in the world *wind* is an important effect on structures, especially in coastal regions and where typhoons occur. Useful information can be found in *Davenport & Dalgiesh, 1970,* and *Ghiocel & Lungu, 1975.*

v [m/s]	1	2	5	10	15
Exceedance time %	70	50	10	1	0.1

Fig. 3/9: Relative exceedance of wind velocities

A characteristic of wind velocities is that small values are frequently observed, but the peak values required for design purposes are rare. The distribution over time of the different wind velocities is given in fig. 3/9. It follows from this figure that moderate wind velocities may have to be considered as an *accompanying* action.

However, wind velocities with peak values lasting from several seconds to about a minute, i.e., gusts, must be classified as a *leading* action. As a first and simplest approximation, these gust values may be estimated using a Gumbel distribution.

The roughness of the terrain, the sheltered parts of buildings, and also alleys cause wind velocities to increase and are some of the factors which considerably complicate the picture. Wind action is above all a dynamic problem, which cannot be easily treated.

Once the distribution of the wind velocity to be taken into account has been determined, the distribution of the wind forces W in [kN/m^2] may be calculated from the velocities V in [m/s] according to:

$$W = C_p \cdot W_0 \quad \text{where} \quad W_0 = \frac{V^2}{1600} \tag{3.16}$$

The wind pressure coefficient C_p takes into account the exposure of the area under consideration and may be taken from codes, e.g., the SIA Standard 261. Internationally this standard has been an important document for determining wind actions. It is recommended to take wind pressure coefficients as mean values, with a coefficient of variation of around 15%. The standard deviation of the model variables M_{Ei} of the wind forces has thereby also been dealt with.

Wind pressure on structures may be positive as well as negative. But often wind suction is regarded as positive in the other direction.

As stated earlier the combination of wind and snow (see, e.g., fig. 1/8) can be dangerous. The geometry of the snow can lead to much greater areas of wind attack and thus to unexpectedly large action effects in the structure.

f) Earthquake

Finally, in the case of *earthquakes,* seismic risk maps for many countries are available. These give the *intensity* for the map's corresponding recurrence period. Typical maps can be found in *Bautechnik, 2005,* and *SwissRe, 2000*.

The position of the epicentre, the geological situation, the ground type, and the structural concept (vibrational behaviour, structural details etc.) are the most important quantities when the *action effects* in the structure resulting from an earthquake are analysed.

In the design of structures, *operating level* and *design level earthquakes* are to be distinguished. Structures and also technical installations have to withstand moderate operating level earthquakes without suffering any damage. Higher design level earthquakes may, depending on the type of structure, cause smaller or greater amounts of damage, providing the building itself does not collapse. Earthquake effects are practically always leading actions.

g) Accidental actions

Finally, there is a big family of actions referred to as accidental actions. Examples are fire, explosion, vehicle impact, avalanches, etc. These actions have in common that they are absent most of the time and in some cases even during the whole life. However, if occurring, they may be quite destructive and lower reliability levels in particular for local damage are accepted.

To describe such actions an occurrence model and probabilistic information on magnitude and location is needed.

Most often the *occurrence* is described by a Poisson process (see chapter 6.33). In its simplest form there is a constant occurrence frequency λ indicating that for instance a certain type of accident will occur once per 10 or once per 100 years.

The *magnitude* is often described by the amount of energy related to the accidental action that may harm the structure. Think of the kinetic energy in the case of car or ship impact and the chemical energy in an explosion or fire. Note that also an earthquake is an example of an accidental action, but because of its high frequency in some countries it is treated separately.

This closes a somewhat rudimentary discussion of the characteristics of different actions on structures. Action is a very broad field requiring detailed investigations. Also, in some areas, intensive research still needs to be carried out. The present brief presentation clearly does not claim to fill this gap.

3.5 Other fields of engineering

This chapter has dealt almost exclusively with structures. This is deliberate, because concentration on a particular range of problems is necessary for clarity's sake. The application of reliability theory is perhaps also most advanced in structural engineering.

It would also be desirable, however, to have information about modelling, models, model variables, basic variables etc. for other fields of civil engineering, i.e., for hydraulic, geotechnical, traffic, and construction engineering. The methods of reliability analysis, as presented in this book, can equally well be applied in the above fields. Mechanical and many other fields of engineering, as well, may profit from the methods described.

To provide the corresponding data and models is, however, beyond the scope of this work and of the authors expertise. It is hoped, therefore, that what is presented here in some detail in relation to structural engineering will be extended to other disciplines, and by others.

4. Reliability Analysis Methods

4.1 Preliminary remarks

Among other requirements structures have to exhibit the following most important features:

- safety
- serviceability

Both requirements related to some predefined period of time (durability) should be achieved by minimum cost (economy). Similar demands are also placed on other technical systems.

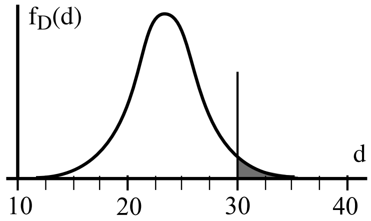


Fig. 4/1: Probability density of D

Each requirement can be formulated using a so-called *limit state condition,* which can generally be written as:

$$G(a_0, X_1, X_2, \ldots X_n) \geq 0 \tag{4.1}$$

The X_i represent the random variables, which describe both the problem and the requirements for a particular basis of assessment (see section 3.21). Random variables are not always physical values like dimensions, strengths, loads, etc., but could well be abstract values, such as the opinion of a group of people about the admissible value of a beam deflection. In fig. 4/1, such an opinion is shown in the form of a probability density: Only the shaded part of all questioned people would be prepared, in the case investigated, to accept a beam deflection of $d \geq 30$ mm. How such variables are derived was discussed in section 2.14.

The so-called *limit state equation* separates the acceptable region from that which is characterised as failure:

$$G(a_0, X_1, X_2, \ldots X_n) = 0 \tag{4.2}$$

Failure is defined by the *failure condition* as:

$$G(a_0, X_1, X_2, \ldots X_n) < 0 \tag{4.3}$$

Of interest in the present connection is the *probability of failure* p_f. This can be written as follows:

$$p_f = P[G(a_0, X_1, X_2, \ldots X_n) < 0] \tag{4.4}$$

There are many methods for determining these failure probabilities or the corresponding reliability indices, each method having its own level of sophistication. Quite often the following subdivision is used, starting with the highest level:

- *Level III:* limit state functions and distribution functions for the random variables are introduced without any approximation; calculations are usually based on Monte Carlo simulation (see 4.2) or straight forward numerical integration;
- *Level II:* the amount of calculation efforts is reduced by adopting well chosen linearization techniques, usually the so called First Order Reliability Method; the degree of accuracy may strongly depend on the details of the problem at hand;

- *Level I:* the variables X_i are introduced by *one single* value only; this value is referred to as the design value. This method does not actually calculate a failure probability but only checks whether some defined target level is attained or not. It is the basis for most design and assessment procedure in every day practice and is referred to as the semi-probabilistic level.

The peculiar difficulty of certain problems in civil engineering lies in the fact that often one has to deal with values that are far away from the mean value. In these areas the probability densities are very small and the obtained results are very dependent on the shape of the so-called "tail" of the distributions.

Besides, one must remember that p_f is a subjective probability. Basically, it is a matter of the *degree of confidence* in the statement that what has been assessed could fail. As discussed in section 2.14, this subjective probability is not an inherent property – e.g., of a bridge – but depends very much on the amount of information available to the person who makes the assessment. Written formally, p_f is a conditional probability, dependent on the state of knowledge of the person doing the assessment.

$$p_f = P[G(a_0, X_1, X_2, ... X_n) < 0 \mid \text{Info}] \qquad (4.5)$$

There are two further limitations to be mentioned here:

- It is assumed in the following that the *variables* in a limit state function are *independent* of each other. Correlations between variables are difficult to determine and considerably complicate the algorithms. This limitation is acceptable, since, if there is uncertainty, both extreme cases – complete correlation and no correlation at all – can be analysed separately, compared, and the differences in the results assessed. Computer programs, however, allow for correlations.
- *Human error* does not enter this kind of analysis. Failure probabilities p_f discussed here are conditional on the assumption that there are no errors in what is analysed. For reducing errors, special strategies measures briefly discussed under section 5.5 would be needed.

This chapter introduces the basic *methods* that are available to calculate, with sufficient accuracy, probabilities of failure p_f under the given assumptions.

4.2 The *Monte-Carlo* method

No method is as easy to understand and as readily compatible to engineering thinking as the *Monte-Carlo* method. And – provided a powerful computer and a suitable program are at hand – none is as adaptable and accurate as this method, although only in the last decade with the emergence of powerful computers has it found increasing application.

With the Monte-Carlo method (or Monte-Carlo simulation) the exact or approximate calculation of the probability density and of the parameters of an arbitrary limit state function of variables

$$G = G(a_0, X_1, X_2, \dots X_i, \dots X_n) \qquad (4.6)$$

is replaced by statistically analysing a large number of individual evaluations of the function using random realisations x_{ik} of the underlying distributions X_i. The index "k" stands for the "k"-th simulation (k = 1, 2 … z) of a set of x_i.

Each set of the k realisations introduced into the limit state function leads to a number

$$g_k = G(a_0, x_{1k}, x_{2k}, \dots x_{ik}, \dots x_{nk}) \qquad (4.7)$$

The resulting z numbers g_k are evaluated statistically according to the rules given in section 2.24.

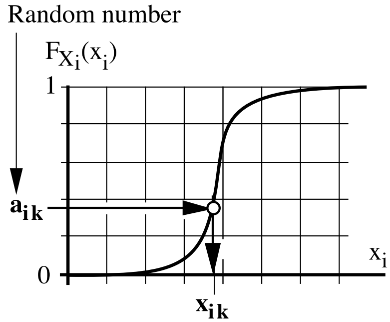


Fig. 4/2: Random number generator

The heart of the method is a *random number generator* that produces random numbers a_{ik} between 0 and 1. Such a number is interpreted as a value of the cumulative distribution function $F_{Xi}(x_i)$ and delivers the associated realisation x_{ik} of the variable X_i.

Now, the number z_0 of failures, i.e., the number of all realisations for which $g_k < 0$, is counted. Thereby p_f can be calculated according to the frequency definition of probability as:

$$p_f = z_0 / z \tag{4.8}$$

In this expression z is the total number of all realisations of G. The greater the number of z_0, the more reliable is the value of p_f. This becomes clear when looking at the coefficient of variation of the probability of failure p_f.

This coefficient, for small p_f, can be written as:

$$v_{p_f} \approx \frac{1}{\sqrt{z \cdot p_f}} \tag{4.9}$$

If a small coefficient of variation is required – e.g., 10% – then for probabilities of failure of, e.g., $p_f = 10^{-4}$ as many as $z = 10^6$ simulations have to be produced.

In addition to counting z_0 and z, g_k could be analysed statistically according to section 2.23, by determining the mean value m_G and the standard deviation s_G (and, if of interest, higher moments too). From these two values the reliability index β (described in detail under 4.33) can be determined, and from it an estimated value for the probability of failure p_f:

$$\beta = m_G / s_G$$

from which

$$p_f \approx \Phi(u = -\beta) \tag{4.10}$$

As can be seen, in this estimate it is assumed that the density of G is normally distributed. If this is not the case the estimate may be quite rough.

In some computer programs the resulting values g_k are continuously presented in a histogram, thus giving immediately an idea of the probability density of the variable G.

In order to reduce the computational effort, a number of so-called "Importance Sampling" methods has been developed, in which the realisations g_k can be focused on the failure region, that is the area of the limit state function where failure is most probable. It is beyond the scope of this work to go into greater detail on these methods.

4.3 The problem G = R – S

4.31 Introduction and example

From the beginning of attempts to solve probability problems, the Monte-Carlo method has been considered, at least intuitively. In practice, however, only in the last 20 years or so has its application become feasible. Before, alternative methods were developed, which, though in some cases less accurate, still today prove to be very powerful and state-of-the-art.

Certain names are attached to the development of such methods. Among the first was *Max Mayer* who formulated his ideas on this topic in 1926 in a book which even today is interesting to read. *A.M. Freudenthal* took up this question in 1947. *Julio Ferry-Borges* has played an important role since 1966 in the further development of both the theory and solution methods, especially during his long and fruitful time as President of the *JCSS*, the *Joint Committee on Structural Safety* founded by the international associations in the field of structural engineering (CEB, CIB, ECCS, FIP, IABSE, IASS, and RILEM).

Other key contributors can be mentioned here in name only: *Ernst Basler, Allin Cornell, Carl Turkstra, Niels Lind, A.M. Hasofer, Ove Ditlevsen, Rüdiger Rackwitz, Michael Baker, Robert Melchers, Henrik Madsen, Armen Der Kiureghian, ...*

The development of such alternate methods is illustrated and discussed in detail here using, in terms of an example, the limit state function G = R – S.

In almost all examples, reference will be made to the following variables, which can be regarded as bending moments in the critical section of a beam:

X_i	Type	μ_{xi}	σ_{xi}
S	N	90	30
R	N	150	20

These variables are either considered as normally distributed values or as rectangular distributions (see also fig. 4/4). In the latter case the variables possess the following limits:

X_i	Type	a_{xi}	b_{xi}
S	R	38.1	141.9
R	R	115.4	184.6

With the aid of these numbers, the fractile values and the conventional deterministic safety factors can be determined.

From the mean values of both variables the "central" safety factor γ_C can then be derived as:

$$\gamma_C = \mu_R / \mu_S = 150/90 = 1.67$$

The so-called safety margin amounts to:

$$m = \mu_R - \mu_S = 150 - 90 = 60$$

Usually, without looking at the type of distribution, the 5% and 95% fractiles are calculated. These are given for a normal distribution by the following expression:

$$x_{5\%} = \mu - 1.65 \cdot \sigma$$
$$x_{95\%} = \mu + 1.65 \cdot \sigma$$

With the numbers used before, this results in

$r_{5\%}$ = 150 – 1.65 · 20 = 117.0
$s_{95\%}$ = 90 + 1.65 · 30 = 139.5

and thus to the nominal safety factor of

$\gamma_N = r_{5\%}/s_{95\%} = 117.0/139.5 = 0.84$

Obviously, this is unacceptable, as safety factors are deemed to be greater than one.

Looking at the rectangular distributions, the 5% and the 95% fractile values are 136.7 and 118.9, respectively. The nominal safety factor, therefore, remains practically unchanged.

These are numbers that attempt to describe safety. But it is difficult to draw conclusions about whether the situation is acceptable. Much more meaningful are failure probabilities, the derivation of which will be discussed below.

4.32 The classical solution

The variables R and S in the limit state function G = R – S are shown, with their respective probability density functions, in fig. 4/3. The picture is valid for any kind of distributions.

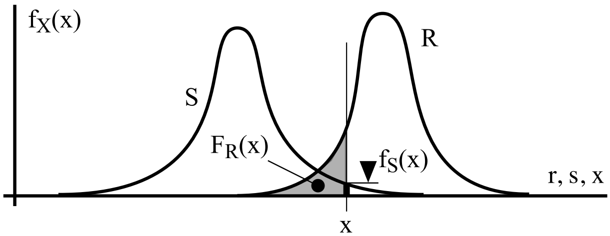


Fig. 4/3: Probability density functions of R and S

In calculating the probability $p_f = P(R - S < 0)$, firstly, the probability that R is smaller than a given value x is of interest. This can be written according to section 2.31 as the value of the cumulative distribution function at the position x:

$$P(R < x) = F_R(x) \qquad (4.11)$$

The probability that S = x is obtained from the probability density function of S at x:

$P(S = x) = f_S(x) \cdot dx$

wherein S = x stands for $x < S \le x + dx$. (4.12)

The probability that both expressions (4.11) and (4.12) are valid is given (see eqn. 2.9) as the product of these values.

Since x may take on any value between – ∞ and + ∞, integration results in:

$$p_f = \int_{-\infty}^{+\infty} f_S(x) \cdot F_R(x) \cdot dx \quad \text{or} \qquad (4.13)$$

$$p_f = 1 - \int_{-\infty}^{+\infty} F_S(x) \cdot f_R(x) \cdot dx \qquad (4.14)$$

This integral, known as the "convolution integral", looks simple (and formally it is), but it can only be solved in a closed form for certain simple cases. Computer programs provide different methods of numerical integration.

The formal integration, f.i., is possible for rectangular distributions. The following example (see fig. 4/4) can easily be understood and will be used in this book at different places.

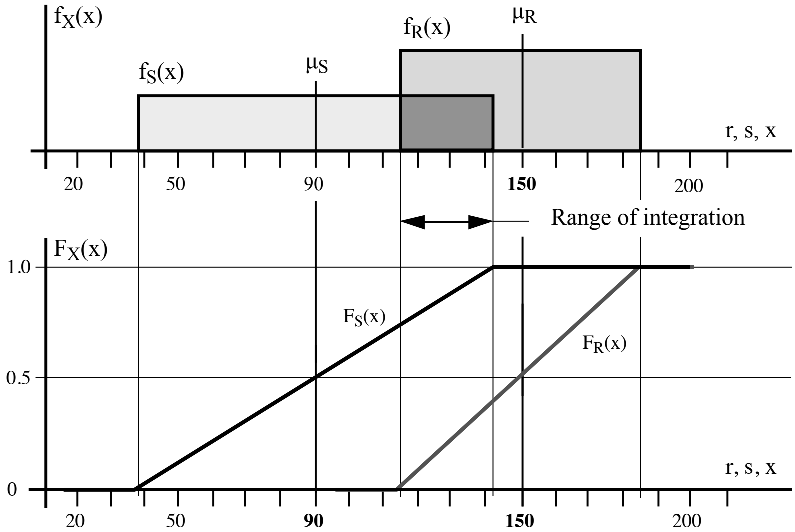


Fig. 4/4: A simple example with two rectangular variables

The following expressions are obviously valid:

$$f_S(x) = \frac{1}{141.9 - 38.1} = 9.63 \cdot 10^{-3} \qquad [38.1 \le x \le 141.9]$$

$$F_R(x) = \frac{1}{184.6 - 115.4} \cdot x - 1.67$$

$$= 14.45 \cdot 10^{-3} \cdot x - 1.67 \qquad [115.4 \le x \le 184.6]$$

Using eqn. (4.13) the integration is as follows:

$$p_f = \int_{115.4}^{141.9} 9.63 \cdot 10^{-3} \cdot (14.45 \cdot 10^{-3} \cdot x - 1.67)\,dx$$

$$= (9.63 \cdot 10^{-3}) \cdot (14.45 \cdot 10^{-3}) \cdot [0.5 \cdot x^2]\Big|_{115.4}^{141.9} - (9.63 \cdot 10^{-3}) \cdot 1.67 \cdot x\Big|_{115.4}^{141.9}$$

$$= 0.049 = 4.9\%$$

This is the probability attached to the statement that $G = R - S < 0$. This value would definitely be too large for structural safety problems. Here, values in the range 10^{-4} to 10^{-6} are usually observed.

For serviceability problems, however, this might be an acceptable order of magnitude. In this context see also section 5.1.

4.33 *Basler,* in the notation of *Cornell*

Basler, 1961 developed the method presented here in the notation of *Cornell*, *1969.* The explanation proceeds, in terms of an example, from the limit state function G = R – S.

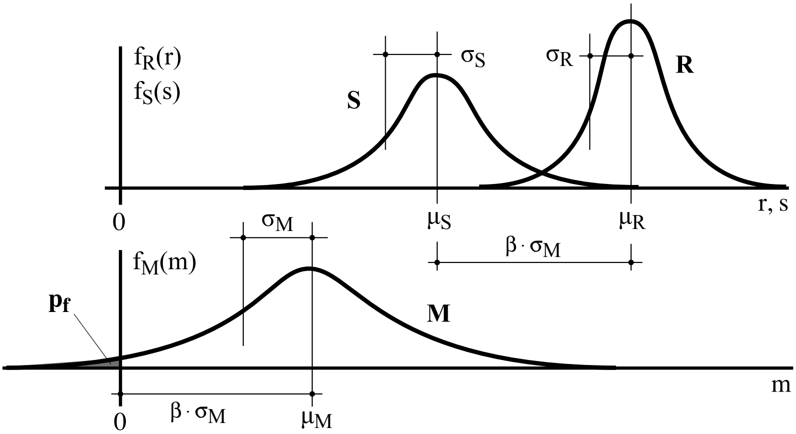


Fig. 4/5: The classical solution via the safety margin M

Fig. 4/5 shows the problem with its variables R, S and M. In fact G is the so-called safety margin M = R – S. As the sum of two variables this margin is also, of course, a variable and is normally distributed if the variables R and S are normally distributed. In this case all the variables may be introduced by their mean and their standard deviation only.

Using the rules of section 2.61 the two first moments of M can be quickly determined. They are:

$$\mu_M = \mu_R - \mu_S \tag{4.15}$$

$$\sigma_M = \sqrt{\sigma_R^{\,2} + \sigma_S^{\,2}} \tag{4.16}$$

From this figure the so-called *reliability index* β can also be seen. It may be determined from the following quotient

$$\beta = \mu_M / \sigma_M \tag{4.17}$$

Expressed verbally: β shows how often the standard deviation of the random variable M may be placed between zero and the mean value of M. The probability of failure is obviously the same as the probability that M is smaller than zero: $p_f = P(M = R - S < 0)$. Assuming normally distributed variables R and S, the failure probability p_f can be read from standard normal distribution tables (see the Appendix under 7.3 for u = – β) as

$$p_f = \Phi\,(-\beta) \tag{4.18}$$

Of special interest are the so-called weighting factors α_i, which show with which weight the corresponding variable participates in the value of the probability of failure. These can be calculated from

$$\alpha_R = \frac{s_R}{\sqrt{\sigma_R^{\,2} + \sigma_S^{\,2}}} \quad \text{and} \quad \alpha_S = \frac{s_S}{\sqrt{\sigma_R^{\,2} + \sigma_S^{\,2}}} \tag{4.19}$$

leading to

$$\alpha_R^{\,2} + \alpha_S^{\,2} = 1 \tag{4.20}$$

As an example, the variables introduced in section 4.31 are used. The numerical analysis delivers the following:

$$\mu_M = 150 - 90 = 60$$

$$\sigma_M = \sqrt{20^2 + 30^2} = 36.1$$

$$\beta = \frac{60}{36.1} = 1.66$$

$$p_f = \Phi\,(-1.66) = 0.049 = 4.9\%$$

$$\alpha_R = \frac{20}{\sqrt{20^2 + 30^2}} = 0.555$$

$$\alpha_S = \frac{30}{\sqrt{20^2 + 30^2}} = 0.832$$

The results are exact for normally distributed variables. For rectangular distributions the result is only approximate. From the α-values it can be seen that the variable S exerts a greater influence on the result than the variable R, due to its larger coefficient of variation.

The procedure of *Basler/Cornell* can be easily developed further into a design condition. The requirement is $\beta \geq \beta_0$, whereby β_0 is representing the "safety level" prescribed in a code. This normally lies, depending on the assumptions, in the range $\beta \approx 3$ to 6. Simply using algebra the following is obtained:

$$\begin{aligned}\mu_R - \mu_S &\geq \beta_0 \cdot \sigma_M \\ &\geq \beta_0 \cdot \frac{\sigma_R}{\sqrt{\sigma_R^{\,2} + \sigma_S^{\,2}}} \cdot \sigma_R + \beta_0 \cdot \frac{\sigma_S}{\sqrt{\sigma_R^{\,2} + \sigma_S^{\,2}}} \cdot \sigma_S \\ &\geq \beta_0 \cdot \alpha_R \cdot \sigma_R + \beta_0 \cdot \alpha_S \cdot \sigma_S\end{aligned}$$

Ordering the terms by R and S leads to:

$$\mu_R - \alpha_R \cdot \beta_0 \cdot \sigma_R \geq \mu_S + \alpha_S \cdot \beta_0 \cdot \sigma_S$$

$$\mu_R \cdot (1 - \alpha_R \cdot \beta_0 \cdot v_R) \geq \mu_S \cdot (1 + \alpha_S \cdot \beta_0 \cdot v_S)$$

This condition may be abbreviated as follows:

$$r^* \geq s^* \tag{4.21}$$

and states the well known and, thus, rather trivial condition that the design value r^* of the resistance must be greater than the design value s^* of the action. The *design values* are the co-ordinates of the so-called *design point*.

In some modern standards r^* and s^* are replaced by r_d and s_d. For clarity reasons in this book the star notation is used.

In principle, the expressions $(1 \pm \alpha_X \cdot \beta_0 \cdot v_X)$ are safety factors. These differ, however, from traditional factors by their transparent structure. The expressions are a function of the desired reliability level β_0; they respond to the size of the standard deviation σ_X or the coefficient of variation v_X of the respective variable X and are moderated by means of the weighting factor α_X.

4.34 Representation as a joint probability density

A different representation of the problem G = R – S is shown in fig. 4/6: The respective two-dimensional joint probability density, as introduced in section 2.54, is again represented as a hump. Its volume is 1 and the contours are concentric curves.

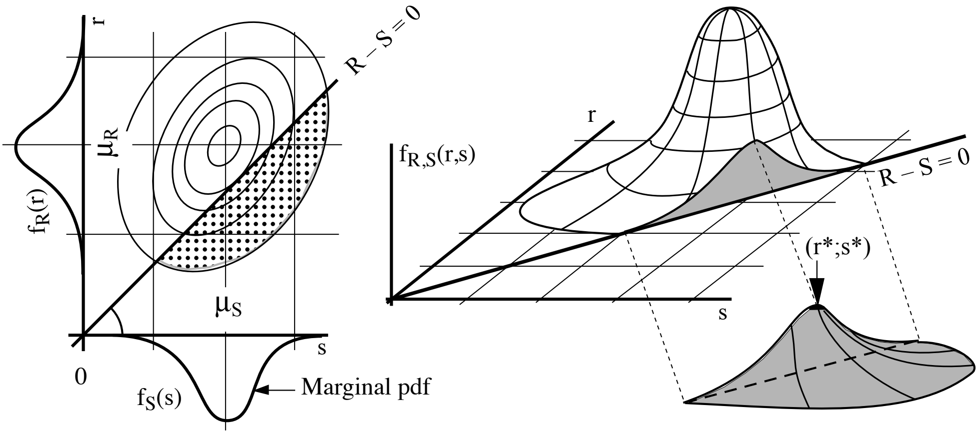


Fig. 4/6: Joint probability density and the design point

In fig. 4/6, R and S are plotted as marginal probability density functions on the r and s axes. The limit state equation G = R – S = 0 separates the safe from the unsafe region, dividing the hump volumetrically into two parts. The volume of the part cut away and defined by s > r corresponds to the probability of failure. The design point (r*;s*) lies on this straight line where the joint probability density is greatest: if failure occurs it is likely to be there.

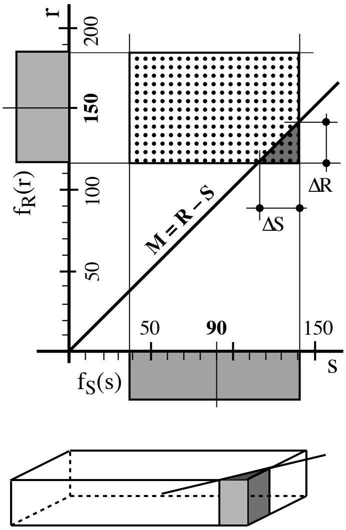


Fig. 4/7: Same example as in Figure 4/4

Turning again to the example of section 4.32, it follows that the joint probability density function is box-shaped (see fig. 4/7). $F_{X,Y}$ is constant within the defined region. Its value, i.e., the height of the "box", can be readily determined, since the volume of the joint pdf is equal to 1. Thus:

$$F_{X,Y} = \frac{1}{(141.9 - 38.1)} \cdot \frac{1}{(184.6 - 115.4)} = \frac{1}{7183} = 139 \cdot 10^{-6}$$

The volume cut away and corresponding to the probability of failure p_f can then be easily calculated. It corresponds to the cut-away triangular area multiplied by $f_{X,Y}$:

$$p_f = V = \Delta S \cdot \Delta R \cdot 0.5 \cdot f_{X,Y} =$$
$$= (141.9 - 115.4)^2 \cdot 0.5 \cdot 139 \cdot 10^{-6} = 0.049$$

This result has already been obtained in section 4.32 and section 4.33.

4.35 The method of *Hasofer* and *Lind*

Following the procedures of *Basler/Cornell* for some more complex limit state functions, *Ditlevsen*, *1973* discovered that the results depend on how the function is formulated. This was called the invariance problem. An important step in solving this problem was made by *Hasofer* and *Lind* (see *Hasofer & Lind, 1974)*. They transformed the limit state function into the so-called standard space. This transformation is shown here for the two normal variables R and S only.

The random variables R and S are transformed and standardised into U_1 and U_2:

$$U_1 = \frac{R - \mu_R}{\sigma_R} \quad \rightarrow \quad R = U_1 \cdot \sigma_R + \mu_R$$

$$U_2 = \frac{S - \mu_S}{\sigma_S} \quad \rightarrow \quad S = U_2 \cdot \sigma_S + \mu_S \tag{4.22}$$

Thus, the new variables have a mean value of 0 and a standard deviation of 1. The greatest rise of the joint probability density hump discussed in the previous section then coincides with the origin of the coordinates. Now that both marginal pdf's are of the same (0;1) character, the figure is axially symmetric, and all contours (points of equal probability) are concentric circles. In the new co-ordinate system the straight line $G = R - S$ no longer passes through the origin. The line transforms into the following:

$$\begin{aligned} G &= R - S \\ &= (U_1 \cdot \sigma_R + \mu_R) - (U_2 \cdot \sigma_S + \mu_S) \\ &= (\mu_R - \mu_S) + U_1 \cdot \sigma_R - U_2 \cdot \sigma_S \end{aligned}$$

The design point $[u_1^*, u_2^*]$ still lies at the highest elevation of the joint pdf above the straight line $G = 0$. Due to the axial symmetry of the hump, the distance from the design point to the origin is equal to the distance marked with β, the so-called HL (*Hasofer/Lind*) reliability index. The further the straight line passes from the origin the greater is β and the smaller is the cut-away volume and thus p_f.

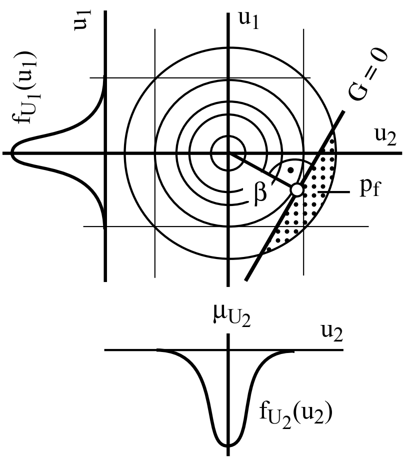


Fig. 4/8: Standardised normal space

A short example should elucidate the procedure. Using the same limit state function G and the parameters of the variables R and S, the transformation results in the following:

$$U_1 = \frac{R - 150}{20} \quad \rightarrow R = 20 \cdot U_1 + 150$$

$$U_2 = \frac{S - 90}{30} \quad \rightarrow S = 30 \cdot U_2 + 90$$

Substituting these into the limit state function G gives

$$G = R - S = (20 \cdot U_1 + 150) - (30 \cdot U_2 - 90)$$

which then defines the straight line by

$$20 \cdot U_1 - 30 \cdot U_2 + 60 = 0$$

The distance of this straight line from the origin of co-ordinates can be quickly found by comparing this equation with the Hessian normal form for representing straight lines:

$$A \cdot x + B \cdot y + C = 0$$

From comparing the coefficients the following numbers are obtained:

A = + 20 B = – 30 C = + 60

From this:

$$\cos\alpha_1 = \frac{A}{\sqrt{A^2+B^2}} = \frac{-20}{\sqrt{20^2+30^2}} = +0.555 \quad \rightarrow \alpha_1 \text{ corresponding to } u_1$$

$$\cos\alpha_2 = \frac{B}{\sqrt{A^2+B^2}} = \frac{-30}{\sqrt{20^2+30^2}} = -0.832 \quad \rightarrow \alpha_2 \text{ corresponding to } u_2$$

$$h = \beta = \frac{-C}{\sqrt{A^2+B^2}} = \frac{-60}{\sqrt{20^2+30^2}} = -1.66$$

Using the table in Appendix 7.3, the probability of failure p_f is obtained:

$p_f = \Phi\,(-1.66) = 0.049 = 4.9\%$

This value is correct in the case of normally distributed variables R and S or U_1 and U_2, respectively, and, for non-normally distributed variables, the value is a good approximation.

The co-ordinates of the design points are obtained as

$$u_1^* = \beta \cdot \alpha_1 = -1.66 \cdot 0.555 = -0.924$$
$$u_2^* = \beta \cdot \alpha_2 = -1.66 \cdot (-0.832) = 1.384$$

or in the r-s coordinate system:

$$r^* = 20 \cdot u_1^* + 150 = 20 \cdot (-0.924) + 150 = 131.5$$
$$s^* = 30 \cdot u_2^* + 90 = 30 \cdot 1.384 + 90 = 131.5$$

All these numerical values have already been observed in the previous sections. The method applied here seems to be more complicated and to bring no advantages. This impression, however, is wrong. The *Hasofer/Lind* method actually permits extension to arbitrary limit state functions and any kind of distribution types as will be shown in the following.

4.4 Extensions of the *Hasofer/Lind* method

What was presented in the previous section is strictly valid only for linear limit state functions and for independent, normally distributed variables X_i. In all other cases the results are – though often quite good – approximations.

The extensions now to be discussed concern the transitions from

- two variables to *many* variables
- linear limit state functions to *non-linear* functions
- normally distributed variables to *any kind* of distribution types.

Because it can account for all these extensions, the *Hasofer/Lind* method is state-of-the-art in reliability analysis.

4.41 Linear limit state functions with many variables

A linear limit state function with several normally distributed variables is generally written in the form:

$$G = a_0 + \sum_{i=1}^{n} a_i \cdot X_i \qquad X_i \text{ given by } \mu_i \text{ and } \sigma_i \tag{4.23}$$

A transformation to the u space is not necessary but is often advantageous when programming for the computer. The algorithm is just a simple extension of the two-dimensional case (see section 4.33) and proceeds as follows (no proof is given):

$$\mu_G = a_0 + \sum_{i=1}^{n} a_i \cdot \mu_i \tag{4.24}$$

$$\sigma_G = \Big(\sum_{i=1}^{n} (a_i \cdot \sigma_i)^2\Big)^{0.5} \tag{4.25}$$

$$\beta = \frac{\mu_G}{\sigma_G} \quad \rightarrow \quad p_f = \Phi(-\beta) \tag{4.26}$$

$$\alpha_i = \frac{\sigma_i}{\sigma_G} \cdot a_i \quad \text{with} \quad \sum_{i=1}^{n} \alpha_i^2 = 1 \tag{4.27}$$

$$x_i^* = \mu_i - \alpha_i \cdot \beta \cdot \sigma_i \tag{4.28}$$

As an example, in fig. 4/9 a simple beam is considered. Actions are a concentrated load F in the middle of the beam and a uniformly distributed load Q (here written in capitals, since it is regarded as a variable). R is the bending resistance in the middle of the beam.

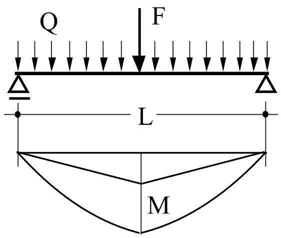


Fig. 4/9: A simple beam

The safety condition is:

$$M = F \cdot \frac{L}{4} + Q \cdot \frac{L^2}{8} \leq R$$

The limit state equation then is:

$$G(X_i) = G(R, F, Q) = R - F \cdot \frac{L}{4} - Q \cdot \frac{L^2}{8} = 0$$

G is no longer a straight line, but represents a plane surface. L is assumed a deterministic quantity of $l = 20$. As dimensions, of course, a consistent set like kN, m, and kNm is implied.

In view of a numerical example the variables are defined as follows:

X	X_{nom}	μ_x	σ_x	v_x
R	2100	2500	250	0.10
F	250	200	50	0.25
Q	10	10	1	0.10

The first column contains nominal values, as, e.g., could be taken from a code. They could be defined as the 5%-fractile for R, the 16%-fractile for F, and a fixed live load Q.

Using these numbers, the traditional safety factors – the nominal safety factor γ_N and the central safety factor γ_C – can be calculated:

$$\gamma_N = \frac{2100}{200 \cdot \frac{20}{4} + 10 \cdot \frac{20^2}{8}} = 1.20 \quad \text{and} \quad \gamma_C = \frac{2500}{200 \cdot \frac{20}{4} + 10 \cdot \frac{20^2}{8}} = 1.67$$

With the previous algorithm the calculation of the reliability index β and of the failure probability is very simple:

$$G = R - F \cdot \frac{20}{4} - Q \cdot \frac{20^2}{4} = 1 \cdot R - 5 \cdot F - 50 \cdot Q$$

$$\mu_G = 1 \cdot 2500 - 5 \cdot 200 - 50 \cdot 10 = 1000$$

$$\sigma_G{}^2 = [(1 \cdot 250)^2 + (-5 \cdot 50)^2 + (-50 \cdot 1)^2] = 127.5 \cdot 10^3$$

$$\sigma_G = 357.1$$

$$\beta = 1000/357.1 = 2.8 \rightarrow p_f = \Phi(-2.8) = 0.0025 = 2.5\ ‰$$

The weighting factors are:

$$\alpha_R = \frac{250}{357.1} \cdot (+1) = +0.70$$

$$\alpha_F = \frac{50}{357.1} \cdot (-5) = -0.70$$

$$\alpha_Q = \frac{1}{357.1} \cdot (-50) = -0.14$$

As may be seen, R and F contribute to the same extent to the probability of failure, whereas here Q is almost negligible. It should also be noticed that the sign of the α-values gives further information on the problem at hand: positive are those α-values whose variables provide safety – i.e., act positively – while the negative sign indicates "hazardous" variables.

The design values of the three variables are:

$$r^* = 2500 - 0.70 \cdot 2.8 \cdot 250 = 2010$$

$$f^* = 200 - (-0.70) \cdot 2.8 \cdot 50 = 298$$

$$q^* = 10 - (-0.14) \cdot 2.8 \cdot 1 = 10.4$$

The results are exact for normally distributed random values. For other distributions of variables they are approximations.

Here is a good place to introduce briefly the notion of *partial safety factors* to be elaborated in section 4.6. These factors relate design values to nominal (or characteristic) values fixed, for instance, in a code. The partial factors for the problem at hand are:

$$\gamma_R = 2100/2010 = 1.04$$

$$\gamma_F = 298/250 = 1.19$$

$$\gamma_Q = 10.4/10 = 1.04$$

It is obvious that these factors depend on how the nominal values are fixed. An important lesson can be learned from this: a partial safety factor may by no means be separated from the respective nominal or characteristic value it belongs to. Had a nominal value of 6 been chosen instead of 10 for the distributed load q, the respective partial safety factor would be 1.73 instead of 1.04, leaving all other numbers including the failure probability unchanged.

4.42 Non-linear limit state functions

In extending the method to non-linear limit state functions the following example is used:

$G = X_1 \cdot X_2 - 10'000$

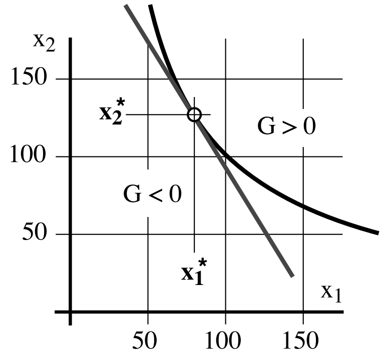


Fig. 4/10: Taylor expansion of non-linear limit state function

In the region of interest – i.e., in the region around the design point – G may be developed as a *Taylor series* (see fig. 4/10 and section 2.61). Linear terms only are taken into account, i.e., higher order terms are neglected.

Using well-known Taylor expansion techniques results in:

$$G \approx G(x_i^*) + \sum_{i=1}^{n} (X_i - x_i^*) \cdot \frac{\partial G}{\partial X_i}\Big|_* + \ldots$$

$$\approx G(x_i^*) - \sum_{i=1}^{n} x_i^* \cdot \frac{\partial G}{\partial X_i}\Big|_* + \sum_{i=1}^{n} X_i \cdot \frac{\partial G}{\partial X_i}\Big|_* \qquad (4.29)$$

The first two terms in eqn. (4.29) correspond to a_0, while the partial differentials at the design point correspond to the a_i in eqn. (4.23).

The limit state function thus linearised may be seen in fig. 4/10 as the tangent at the design point. The "*" in eqn. (4.29) draw attention to the fact that the partial differentials have to be calculated at the design point. The linearisation allows tracing back to what was described in section 4.41. The co-ordinates x_i^* of the design point, however, are not yet known but are determined iteratively.

The iterative process is as follows:

1) Approximate limit state function $G = G(X_1, X_2 \ldots X_n)$ by $G = a_0 + \sum_{i=1}^{n} a_i \cdot X_i$

2) Determine $a_i = \frac{\partial G}{\partial X_i}\Big|_*$ and $a_0 = G(x_i^*) - \sum_{i=1}^{n} a_i \cdot X_i$

3) Estimate x_i^* (e.g., start with μ_i)

4) Calculate
   μ_G =
   σ_G = According to eqn. (4.24) to eqn. (4.28)
   β = for linear
   α_i = limit state functions
   x_i^* =

5) Compare x_i^* with the values of 3):
   - is the approximation good enough → 6),
   - if not satisfactory → 3) with the last determined x_i^*

6) Calculate $p_f = \Phi(-\beta)$.

Naturally, the results are an approximation, even in the case of normally distributed variables, since the limit state function is non-linear.

The approach to adopt is shown using the example presented in fig. 4/10. The variables are defined by $X_1 = N(150; 20)$ and $X_2 = N(90; 30)$.

1) $G \;=\; X_1 \cdot X_2 - 10'000$ is approximated by $G = a_0 + a_1 \cdot X_1 + a_2 \cdot X_2$

2) $a_1 \;=\; \dfrac{\partial G}{\partial X_1}\Big|_* = X_2\big|_* = x_2^*$

$a_2 \;=\; \dfrac{\partial G}{\partial X_2}\Big|_* = X_1\big|_* = x_1^*$

$a_0 \;=\; x_1^* \cdot x_2^* - 10'000 \; - 2 \cdot x_1^* \cdot x_2^* = - 10'000 \; - x_1^* \cdot x_2^*$

$G \;\approx\; - 10'000 \; - x_1^* \cdot x_2^* + x_2^* \cdot X_1 + x_1^* \cdot X_2$

3) Start with first estimate of x_i^*, e.g., with the mean values:

$x_1^* \;=\; 150$

$x_2^* \;=\; 90$

4) $\mu_G \;= - 110'000 \; - 13'000 + 90 \cdot 150 + 150 \cdot 90 = 3'500$

$\sigma_G \;=\; \sqrt{(90 \cdot 20)^2 + (150 \cdot 30)^2} = 4'847 \;\rightarrow \beta = \dfrac{3500}{4847} = 0.722$

$\alpha_1 \;= \dfrac{20}{4'847} \cdot 90 = 0.371$

$\alpha_2 \;= \dfrac{30}{4'847} \cdot 150 = 0.928$

$x_1^* \;=\; 150 - 0.371 \cdot 0.722 \cdot 20 \;=\; 144.6$

$x_2^* \;=\; 90 - 0.928 \cdot 0.722 \cdot 30 \;=\; 69.9$

The differences between these values and the first estimates are considerable, and a further run with the last calculated values as start values would be necessary. This second run and, possibly, additional ones are left to the reader. The final result is:

$\beta \;=\; 0.744$

$x_1^* = 145.5$

$x_2^* = 68.7$

From β the probability of failure can be determined. This determination corresponds to 6) above and, using standard normal distribution tables, results in

$p_f = \Phi(-\beta) = \Phi(-0.744) = 0.23$

In reading from the tables, the assumption is made that G is normally distributed. As this, generally, is not the case, the result is an approximation.

In the above method only the *first* order term of the Taylor series was considered. It, therefore, is designated as the *"**F**irst **O**rder **R**eliability **M**ethod"* or, abbreviated, the *FORM* method. FORM is not restricted to two variables. The tangent to the limit state function in the case of three variables transforms into a tangent plane and for many variables into what is called a tangent hyperplane.

If also the *second* order term of the Taylor expansion is included in the analysis then in fact the limit state function is approximated by a tangent hypersurface that also fits the curvature of the limit state function in the design point. The respective method is consequently called the *"Second Order Reliability Method" SORM*.

4.43 Non-normally distributed variables – Tail approximation

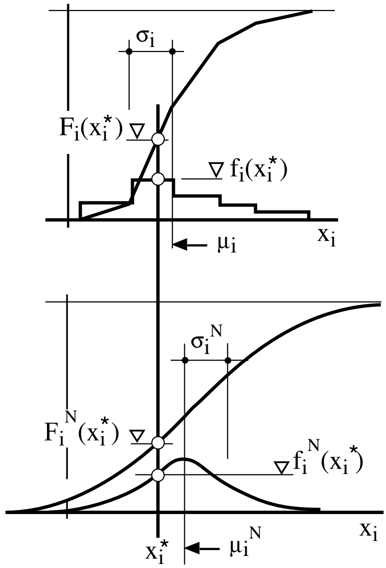


Fig. 4/11: Tail approximation ...

The extension discussed here is the transition from normally distributed variables to arbitrary, mathematically defined distributions or histograms.

The tail approximation approach is based on the idea of replacing arbitrary distributions by equivalent normal distributions at the design point (*Rackwitz*, *1977)*. Referring to fig. 4/11, equivalence exists if such an equivalent normal distribution at the design point x_i* simultaneously fulfils $f_i^N(x_i^*) = f_i(x_i)$ and $F_i^N(x_i^*) = F_i(x_i)$. Non-identical, however, are the mean and the standard deviation. Both these parameters have to be determined first.

As the "tail" of the distribution, which is to be replaced by a normal distribution is of concern here, this approximation is appropriately called the *tail approximation*. It appears that, for each new value of x* in the iteration process, a new equivalent normal distribution has to be determined. This is only feasible with the aid of a computer.

The parameters $\mu_i^N(x_i^*)$ and $\sigma_i^N(x_i^*)$ of the equivalent normal distribution are obtained analytically from the equations:

$$\mu_i^N(x_i^*) \quad = \quad x_i^* - \sigma_i^N(x_i^*) \cdot \Phi^{-1}[F_i(x_i^*)]$$

$$\sigma_i^N(x_i^*) \quad = \quad \varphi\{\Phi^{-1}[F_i(x_i^*)]\} / f_i(x_i^*)$$

If the reader is not familiar with the term $\Phi[F_i(x_i^*)]$ he/she may visit the Appendix, section 7.3 and have a look to the top of the table.

The graphical determination of the parameters of the equivalent normal distribution makes use of normal probability plotting paper (see section 2.43). The given distribution $\{F_i(x_i)\}$ is plotted on this paper and the tangent is constructed on the resulting curve at the design point x_i*. Such a tangent obviously has at point x_i* the same function value $F_i(x_i^*)$ and the same slope $f_i(x_i^*)$ as that of the corresponding curve for the given distribution. Since the tangent is a straight line on normal probability paper, it represents the equivalent normal distribution searched. The mean value $m_i^N(x_i^*)$ can immediately be read off at 0.5. The associated standard deviation $s_i^N(x_i^*)$ is determined by reading the values at 0.159 and 0.841 and halving the difference.

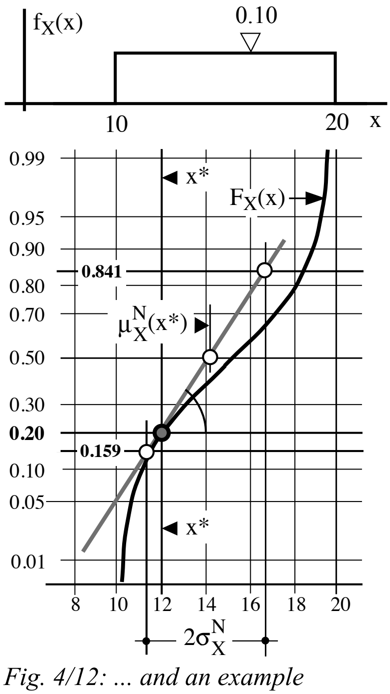


Fig. 4/12: ... and an example

Fig. 4/12 shows, on normal probability paper, the rectangular distribution R(10;20) and for x* = 12 the equivalent normal distribution as the straight line.

The values f(x*) = 0.1 and F(x*) = 0.2 can be read from the figure. The standard deviation of this rectangular distribution, according to Appendix 7.2, amounts to:

$$\sigma_X = (20-10)/\sqrt{12} = 2.89$$

In the table for the standard normal distribution (see Appendix 7.3) for $\Phi^{-1}[0.2]$ the value – 0.842 can be found. Directly alongside $\varphi(-0.841)$, the value 0.2798 is read.

Thus, finally, the parameters of the equivalent normal distribution at x* = 12 are calculated as follows:

$$\mu^N(x^*) = 12 - 2.8 \cdot (-0.841) = 14.36$$

$$\sigma^N(x^*) = (1/0.1) \cdot 0.2798 = 2.798$$

The graphical method delivers the same values, but naturally within the limits of accuracy in reading the values.

4.44 Working in the U-space for non-linear and non-normal variables

In section (4.35) the random variables R and S were transformed into the standard normal variables U_1 and U_2, having zero mean values and unit standard deviations. This is referred to as a transformation of the X-space (physical quantities) to the U-space (mathematical variables). The limit state function G = R – S was simply linear.

It has some advantages to make use of this transformation to the U-space also in the case of non linear limit state functions and non normal variables.

The advantages are:

- all reliability problems are presented in a uniform way;
- there is no further need for a tail approximation as presented in 4.43: it is implicitly already incorporated in the transformation;
- in the U-space the reliability index β may be defined as the shortest distance from the origin to the (curved) limit state; this means that a large toolkit of operational mathematical methods becomes available to solve a general reliability problem.

If all X are independent the transformation from X_i to U_i is described by the equation:

$$\Phi(U_i) = F_i(X_i) \tag{4.30}$$

where F_i is the distribution function of X_i (may be anything) and Φ the distribution function of a standard normal variable (see 7.3).

If X_i are already normal the equation (4.30) reduces simply to $X_i = \mu_{X_i} + \sigma_{X_i} \cdot U_i$ as before. So, in a formal sense, the calculation of

$$P[G(X_1, X_2, ...) < 0]$$

is replaced by

$$P\left[G(F_1^{-1}\Phi(U_1), F_2^{-1}\Phi(U_2), ...) < 0\right] \tag{4.31}$$

The FORM procedure then follows exactly the lines of 4.35, however with U instead of X as basic variables. Note that an original linear LSF becomes nonlinear if non normal variables are involved.

4.45 Remarks on correlated variables

Any correlation between the variables X_i and X_j can be obtained from the associated correlation coefficients $r_{i,j}$ and $\rho_{i,j}$, respectively (see section 2.52). Such correlations can both increase and reduce the probability of failure p_f.

The foreman on a building site, e.g., can act in a positively correlating manner. If the building site is properly managed, then probably the concrete quality will be good as well, the upper steel bars will probably be correctly positioned and not sag down. All these forms of good care may be correlated to one single factor, the capable foreman, and, thus, all may act in the same direction reducing the probabilities of failure. Of course, the opposite might also be true. Similarly, a good working atmosphere in an engineering office can positively influence all the activities within that office.

Negatively correlated are, e.g., snow and live loads on a bridge. If a lot of snow lies on the bridge it is hardly possible to accommodate a high live load. The correlation acts in a reducing way on the probability of failure.

In many cases, however, there are dependencies which can more or less cancel each other out. The cross-sectional dimensions of small rolled steel sections exhibit greater coefficients of variation; the strengths, however, due to the intensive rolling process, are greater as well.

Basically it is no problem to extend the previously discussed computational methods such that they are also capable of dealing with correlations. Mathematically it is a question of rotating the $(X_i;X_j)$ co-ordinate system such that the correlation coefficient between these two variables becomes zero (looking to fig. 2/16, e.g., some rotation is necessary).

In the attempt to keep theory and text short and easy to understand, in this book correlations between variables are not considered, for the following reasons:

- Accommodating correlation renders the formulae and the algorithms more complicated and the reliability analysis more tedious.
- Correlation coefficients are often unknown and, certainly, difficult to estimate. For the user of a computer program the input is also more tedious because he/she is forced to input correlation coefficients between all variables into a correlation matrix. This can be quite elaborate in the case of many variables.
- In the absence of adequate information about correlations the user most probably will input a correlation matrix consisting essentially of ones and zeroes: "+1" on the diagonal and where a considerable positive correlation is expected, "–1" where some important negative correlation is assumed, and "0" where a correlation between variables seems to be absent. If this were the case, the procedure can be simplified, in that the limit state function is adapted, by replacing dependent variables by corresponding regressions to the variables adopted. In doing so even non-linear regression can be introduced.

It must be pointed out, however, that correlations cannot always be handled in the above sense. And, certainly, there are cases where even small correlations may affect the results considerably.

If the limitation introduced here quite deliberately is not acceptable to the reader then he/she should consult the available literature (see, e.g., *Hohenbichler & Rackwitz, 1981; Thoft-Christensen & Baker, 1982; Melchers, 1999; Ditlevsen & Madsen, 1996)* or try suitable computer programs.

4.46 The use of computer programs

It is clear that computations of the type described above cannot be carried out by hand. Computer programs have been developed for this purpose, some of which are commercially available. Among the latter is the computationally very efficient and complete program package *STRUREL*. Completeness, in turn, asks for quite some knowledge and insight from the user.

A more educational type of program is *VaP* and its downsized version *FreeVaP* available free of charge. It includes all the computational methods discussed previously as well as other established methods.

The Appendix 7.5 presents a short description of the above mentioned programs and extends to two other programs, e.g., *SAPOS* and *PROB2B*.

4.5 Time dependency

4.51 Failure probability and failure rate

Loads and resistances are not constant in time but fluctuate or change gradually. Examples of load fluctuations have been given in section 3.4. Changes in structural resistance may occur under the degrading influence of mechanical stresses and/or physical-chemical agencies. When taking these load fluctuations and aging effects into account, the probability of failure depends on the period under consideration: the longer the period, the higher the failure probability.

Next to the failure probability for a given period t (e.g. the design working life), the calculation of the probability of failure per unit of time, also called the failure rate (e.g. the annual failure probability) is necessary. The failure rate is equal to the first derivative of the total failure probability with respect to time. Vice versa, the total probability may be found as the integral of the failure rate.

The failure rate mentioned above is often referred to as the *unconditional* failure rate as opposed to the so called *conditional* failure rate, which is defined as the failure rate at time t, given that no failure occurred before that time. It results from the unconditional failure rate by dividing by the probability of survival:

$$r(t) = (dP_f/dt) / (1 - P_f) \tag{4.32}$$

where $P_f(t)$ is the failure probability for the interval (0,t). For small failure probabilities the numerical difference between the conditional failure rate r(t) and the unconditional failure rate dP_f/dt of course may be neglected.

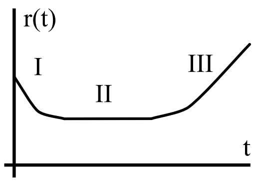


Fig. 4/13: Bath-tub curve

Fig. 4/13 shows an example of a (conditional) failure rate as a function of time having the shape of a bath-tub. In the first period of the existence of the structure the failure rate goes down. This is the result of the fact that after successfully surviving a number of loads, the likelihood to have a low quality structure decreases. This reduction is relatively important if the scatter in the resistance is relatively large. In the horizontal part of the bathtub curve, failure due to calamities like fire and explosion dominate. Finally, the raising at the end of the curve indicates the presence of aging mechanisms.

The calculation of the third part may best be performed by starting from a well-defined physical description of the degradation process. For mechanisms like fatigue in steel structures or carbonation in concrete structures, these models are well available. Just incorporate these models into the limit state formulation as before. Another, less sophisticated, but sometimes useful approach, is to start from mathematical models and then tuning the parameters on the basis of general observations. Quite popular is the use of a so-called condition *classification system,* like for example, class 1 = "good", class 2 = "moderate" and class 3 = "bad". More quantitative descriptions are possible. The jump times, where a change from one condition class to another takes place, usually is, for convenience, modelled as a Markov process, but of course other options are possible.

4.52 Inspection and maintenance

If there is an uncertain deterioration process in a structure, usually a maintenance programme in combination with monitoring or inspection is set up. Such a system may lead to reductions in the otherwise increasing conditional failure rate (see fig. 4/14). Note that in the case of inspection the combination with maintenance in case of unfavourable inspection results is essential in order to have a drop down in the conditional failure rate. Note also that the failure probability for the total period never will go down.

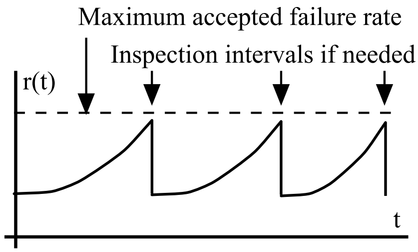


Fig. 4/14: Inspection and maintenance

In a formal sense the, failure probability for a period t may be written as:

$$P_f(t) = P[_{min}G(\tau)<0] \quad \text{for} \quad 0 < \tau < t \tag{4.33}$$

where the time τ may be present explicitly or implicit via for instance time dependent loads. In some cases this formulation may be used directly in practical calculations. More often, however, the so-called outcrossing approach is being followed.

$$P_f(t)=1- \exp[(1 - \nu(\tau)\cdot d\tau)] \tag{4.34}$$

This expression is exact with $\nu(\tau) = r(\tau)$. However, in the outcrossing approach $\nu(\tau)$ is taken to be the so-called outcrossing rate, defined as:

$$\nu(\tau) = P[G(\tau) > 0 \cap G(\tau + \Delta\tau) < 0] / \Delta\tau \quad \text{for limit } \Delta\tau \Rightarrow 0 \tag{4.35}$$

In this way an approximation is obtained. In order to increase the accuracy the calculations may be made first conditional upon completely time independent parameters and integrated later.

Note finally that if the failure rate is constant, say $\nu(\tau) = \lambda$, and the period t small, the following expression is valid

$$P_f(t) = 1- \exp[- \int_0^t \nu(\tau)\cdot d\tau] = 1- \exp(- \lambda\cdot t) = \lambda\cdot t \tag{4.36}$$

Actually, this parameter λ is the main reliability characteristic of many devices in mechanical and electrical engineering. In fact it is in the value of the horizontal branch in the bathtub curve; sometimes a linear second branch is added to simulate aging (see *Karadeniz, 2006* and *Klutke et al., 2016)*

4.6 Deriving partial safety factors

4.61 Design formats

There are, traditionally, quite a number of design formats. One still prevailing in many areas is the *allowable stress format* requiring that maximal stress in a structure be less than allowable values described and numerically fixed in codes and standards for different materials. Written in the form of a safety condition, this format reads:

$$_{allow}\sigma \geq {}_{max}\sigma. \tag{4.37}$$

This format is being replaced now in many new codes by the so-called *load/resistance-factor (LRF),* or *split factor format*. Here, two safety factors, applied to the resistance and to the load side, respectively, serve to keep load effects well below what a structure can resist. The format reads as follows:

$$\phi \cdot R = R/\gamma_R \geq \gamma_S \cdot S \tag{4.38}$$

wherein γ_S and γ_R are load and resistance factors, respectively, and larger than 1, while, alternatively, $\phi = 1/\gamma_R$ is a resistance reduction factor as included in e.g., North American codes.

The so-called *partial factor format* applies factors to all relevant design parameters, e.g., different factors for dead and live loads, different strengths, for instance, in the resistance of a reinforced concrete beam. The format is best explained using the so-called design values x_d, defined by multiplying characteristic values x_k by partial factors γ_k. Both values and factors are defined for loads and strengths variables. The format reads

$$R(r_{di}) \geq S(s_{d_i}), \tag{4.39}$$

where R(...) and S(...) represent functions of design values, which are defined, as stated above, by

$$\begin{aligned} s_{d_i} &= \gamma_{S_i} \cdot s_{k_i} \quad \text{and} \\ r_{d_i} &= r_{k_i} / \gamma_{r_i} \end{aligned} \tag{4.40}$$

Often, in addition, a model factor is introduced.

The probabilistic format advocated in this book reads

$$p_f = P(G = G(X_i) \leq 0) \leq {}_{adm}p_f = \Phi(-\beta_0), \tag{4.41}$$

where β_0 is the target reliability index.

As explained in section 4.4, this condition is equivalent to

$$G = G(x_i^*) \geq 0 \tag{4.42}$$

where, according to eqn. (4.28)

$$x_i^* = \mu_i - \alpha_i \cdot \beta_0 \cdot \sigma_i \tag{4.43}$$

are the design values derived for instance using the Hasofer/Lind method in its various extensions as explained in section 4.4. It is obvious from eqn. (4.40) and (4.43) that the differentiation between load and resistance variables disappears, as well as does the need to allocate variables to the left or to the right side of the safety condition.

4.62 Partial factors

The similarity between expressions (4.40) and (4.43) is evident. This similarity may be used to derive partial safety factors. The following is valid:

$$x_{d_i} = x^*$$

$$\gamma_i \cdot x_{k_i} = \mu_i - \alpha_i \cdot \beta_0 \cdot \sigma_i$$

From the comparison, the partial factor relevant for the variable X_i can be derived as

$$\gamma_i = (\mu_i - \alpha_i \cdot \beta_0 \cdot \sigma_i) / x_{k_i} \qquad (4.44)$$

Introducing the coefficient of variation $v_i = \sigma_i/\mu_i$ into the above the expression reads

$$\gamma_i = \mu_i \cdot (1 - \alpha_i \cdot \beta_0 \cdot v_i)/x_{k_i} \qquad (4.45)$$

All important characteristics of the respective variable come into play in eqn. (4.45), e.g., the mean μ_i of the variable, its coefficient of variation v_i, a characteristic value x_{k_i} of the variable (freely chosen, or prescribed in a code), the relative importance of the variable within the safety condition (expressed by α_i), and, finally, the target reliability level expressed by β_0.

The above is valid for normally distributed variables. As the α-values for "dangerous" variables become negative, the term in brackets becomes larger than 1. The inverse is true for "favourite" variables.

Since the α-values are dependent on the specific design situation under consideration, the partial factors to be applied for a variable depend on that same situation. This fact is unacceptable as for practical reasons constant partial factors are a necessity.

4.63 Linearisation

In order to arrive at a simple semi-probabilistic design format, the weighing factors α may be fixed to some values that, for the more frequent design situations, are checked to give results on the safe side of the target reliability. For more information on load combination factors see *Braestrup, 2012*.

So, e.g., for the most important *resistance variable* in a design situation, $\alpha_R = 0.8$ is often chosen, while for all other resistance variables, rather small values are chosen or even $\alpha_{R_i} = 0$ (resulting in the mean or nominal values).

For the most important *load variables*, e.g., the leading or dominant variable in a hazard scenario (or load combination), likewise, $\alpha_S = 0.7$ may be fixed. For all non-dominant load variables, smaller α-values (usually $\alpha_S = 0.28$) are chosen. Of course, the sum of all α_i^2 must be larger than 1 in order to be on the safe side.

When combining independent time variant loads (see section 3.43) one may even use further reductions to account for the fact that peak values in general do not coincide. In this way, a partial factor code format calibrated to a predefined target reliability may be developed. It is beyond the scope of this book, however, to delve deeper into the many problems related to such a development.

4.7 Elementary approach to the reliability of systems

4.71 General remarks

The probability of failure p_f, for which computational methods were presented in the foregoing sections, generally characterise only the reliability of an *element* of a complete *system*.

Each system – e.g., a structure – comprises as a rule many elements, whose individual or combined failure can lead to collapse. A simple beam can, e.g., fail in bending, in shear or due to failure in the region of the supports. For statically indeterminate systems, usually only combinations of failing elements lead to a failure of the system.

What applies for a structural system consisting of elements is valid not only for other systems, such as traffic systems, water supply systems, and logical sequences of operations, but also for organisational structures, in which the inherently fallible human being plays the role of unreliable "elements". Therefore, it is necessary at least to glance at the elementary principles of the reliability theory of systems. For more details the reader should consult the literature (see, e.g., *Ditlevsen & Madsen, 1996).*

It should be observed that the diagrams presented below are *not physical but rather logical* models. The formulae described in the following may give the impression that parallel systems are more reliable than series systems. This is true for the function diagrams considered but it would in general be a false conclusion. Series systems are very useful if one, e.g., wants to be sure that in a pipe network, if a valve fails *no* water escapes. Then, naturally, several valves in series are needed in order to increase the reliability of the system. The opposite would be true if the *flow* of the water should be ensured. In that case a parallel system of valves would be appropriate.

4.72 Definitions

a) Single element

In the graphical representation of systems, elements may be represented by boxes, which are connected together by lines to form the systems. Input and output are marked by arrows.

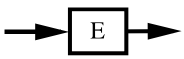


Fig. 4/15: Single element

For a single element E, in respect to the reliability or the probability of failure, the following apply:

$$r = 1 - p_f \quad \text{resp.} \tag{4.46}$$

$$p_f = 1 - r$$

For the reliability of a complete system a capital letter R is written, while for the probability of failure of systems the corresponding capital P_f is used.

b) Series systems

In a series system the individual elements are connected in series in regard to their function (see fig. 4/16). The failure of a single element causes the failure of the whole system. As a simple example, consider a chain consisting of many links. If the weakest link breaks the chain fails. The least reliable link determines the reliability of the chain. Statically determinate systems are series systems. The failure of *one* support, e.g., or of *one* member of a statically determinate truss, leads to the collapse of the whole structure.

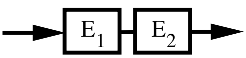


Fig. 4/16: Series system

The reliability R of a series system is given by the probability that neither E_1 nor E_i nor E_n, nor any of its elements will fail. This probability is given by:

$$R = (1 - p_{f1}) \cdot (1 - p_{f2}) \cdot \ldots \cdot (1 - p_{fn}) = \prod_{i=1}^{n}(1 - p_{fi}) \qquad (4.47)$$

Be aware that *statistical independence* of the participating elements is a condition.

The probability of failure P_f of a series system is given by the complement of the reliability of the system:

$$P_f = 1 - \prod_{i=1}^{n}(1 - p_{fi}) \approx \sum_{i=1}^{n} p_{fi} \qquad (4.48)$$

The summation approximation is valid for small probabilities of failure p_{fi}. Obviously, the probability of failure P_f of a series system clearly increases with the number of elements and largely depends on the probability of failure of its most unreliable element.

For example, if a truss has 7 members, each with a probability of failure of $p_f = 10^{-2}$, and if no further types of failure are considered, the probability of failure of the truss amounts to:

$$P_f = 1 - (1 - 10^{-2})^7 = 1 - (0.99)^7 \approx 7\%$$

If for 6 of the 7 members $p_f = 0.01$, but for the seventh $p_f = 0.1$, the failure probability is $P_f \approx 15\%$ and is essentially influenced by the probability of failure of the weakest member.

If all elements of a series system are *perfectly correlated,* e.g., all are produced from the same batch of material, then:

$$P_f = \max\,[p_{fi}]. \qquad (4.49)$$

Thus the probability of failure P_f of a series system lies within:

$$\max\,[p_{fi}] \le P_f \le 1 - \prod_{i=1}^{n}(1 - p_{fi}) < \sum_{i=1}^{n} p_{fi} \qquad (4.50)$$

To estimate conservatively the probability of failure of a series system, P_f may be calculated from eqn. (4.48).

c) Parallel systems

In a parallel system the elements of the system are connected in parallel in regard to their function (see fig. 4/17). The failure of such a parallel system requires that E_1 *and* E_i *and also* E_n fail. Only when *all* elements fail does the system fail. In terms of probability theory:

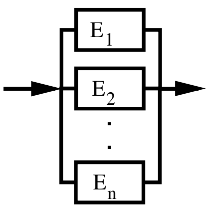


Fig. 4/17: Parallel system

$$P_f = p_{f1} \cdot p_{f2} \cdot \ldots \cdot p_{fn} = \prod_{i=1}^{n}(p_{fi}) \qquad (4.51)$$

Again *statistical independence* of the elements is a condition.

As an example: The drinking water supply of a district is guaranteed by two independent supplies. Each by itself can supply the area with sufficient water. Experience shows, however, that one pipe fails on average for about 2 days per year. The probability of failure per day and pipe is thus:

$$p_f = \frac{2}{365} \approx 0.0055$$

The probability that on the same day both supplies simultaneously fail is:

$P_f = p_{f1} \cdot p_{f2} = 0.0055^2 = 3 \cdot 10^{-5}$

Thus, this probability of failure is very small. It is based on the assumption, however, that both supplies are statistically independent and also that both supplies do not fail due to a common cause. This would, f.i., be the case if both supplies are fed by the same electric power supply.

If all elements are *completely correlated*, then:

$$P_f = \min [p_{fi}] \tag{4.52}$$

i.e., the probability of failure P_f of parallel systems cannot be greater than the probability of failure p_f of the most unreliable element of the system.

The probability of failure P_f of a parallel system, therefore, lies within the following limits:

$$\prod_{i=1}^{n}(p_{fi}) \leq P_f \leq \min [p_{fi}] \tag{4.53}$$

For the case of complete negative correlation, the lower bound becomes zero.

To estimate conservatively the probability of failure P_f, it is assumed that

$$P_f \approx \min [p_{fi}] \tag{4.54}$$

d) Mixed systems

In practice, mixed systems are usually found, i.e., systems which exhibit series connections with parallel elements. Fig. 4/17 illustrates such a case.

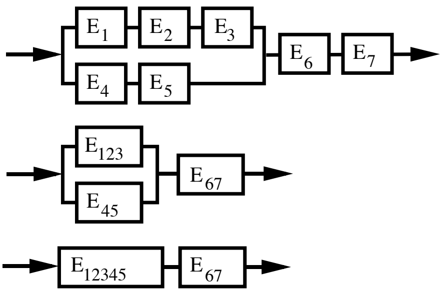


Fig. 4/18: Mixed system and its transformation into a seres system

The reliability or, conversely, the probability of failure of mixed systems can be determined by a stepwise reduction of the parallel elements or the elements connected in series to simple systems, as shown in fig. 4/18. The probabilities of the elements are calculated according to the rules given above. It is left to the reader to apply the rules and thus to determine the probability of failure of the system shown on top of fig. 4.18.

As a check: Assuming that all elements are statistically independent and $p_{fi} = p_f = 0.02$, the value $P_f = 0.042$ results. For a complete correlation using the conservative equations given above, the value $P_f = 0.08$ is found.

Now, which is the element that governs the probability of failure and thus represents the most unreliable part of the system, consequently, deserving the greatest attention in relation to safety measures?

4.73 Structural systems

a) Introduction

A building structure, as illustrated in Fig 4/19, is in the first place an assembly of physical components like beams, columns, slabs, joints, etc. In some cases the relation with logical elements as treated in the previous sections is quite obvious. It is clear that the failure (collapse) of a single component in a statically determinate structure will cause failure (collapse) of the total system. So the three systems to the left are examples of series systems.

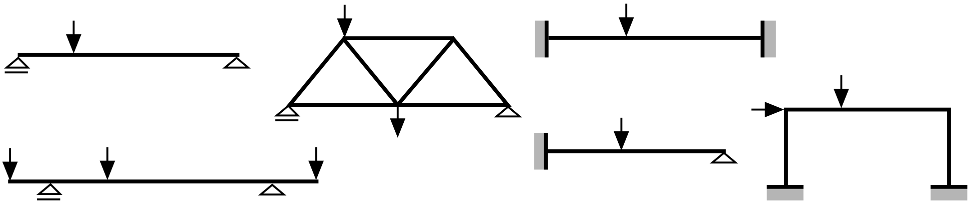


Fig. 4/19: Various structural systems

In statically indeterminate structures (see right side of fig. 4/19) yielding of a single cross section usually does not lead to the immediate collapse of the total structure. Provided that the material in the vicinity of the critical cross section has a sufficient degree of ductility, a further stable increase of the load is possible.

However, statically indeterminate systems really functioning in a parallel sense are very rare. This becomes clear when considering, e.g., a beam built-in at both ends. This system, on first sight, might well be considered as a parallel system as three sections must fail in order for the system to fail. Looking more closely at the problem, it becomes clear, however, that the carrying capacity of the three sections are almost completely correlated with the effect that the probability of failure of the system is reduced to the failure probability of one element.

In addition it must be stated that the question of brittle versus ductile failure of an element in a parallel system is of prime importance. While a ductile element (e.g., a section allowing for some plastic rotation under the ultimate bending moment) may continue to carry load until the other elements of the system yield, a brittle element stops carrying its share of the load leaving the rest of the elements with even more load.

Therefore, when it comes to analysing probabilities of failure of statically indeterminate structural systems, it is appropriate to consider the element with the largest failure probability as the one dominating the problem.

One may also find examples of logical parallel systems in other fields of structural engineering. The application of such systems usually involve aspects like the success or failure of inspections, the success or failure of fire alarms, etc.

b) An introductory example

We will confine ourselves here primarily to the ideal ductile structure with sufficient deformation (rotation) capacity. Note that in reality this requires a set of conditions that also should be taken into account in the reliability analysis.

Consider the statically indeterminate beam with l = 10 m span shown in fig. 4/20. The beam is loaded by a concentrated load F in the centre and has an ideal elastic plastic cross sectional behaviour. Let the force F be a normally distributed random variable with F = N(100;20) and the plastic moment along the whole beam a normally distributed random variable M_p = N(300;30). Units in kN and m. The resistances in all cross sections are considered as fully dependent.

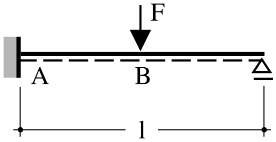


Fig. 4/20: Beam built-in at left support

According to the elasticity theory the bending moments in the critical points A and B are:

$M_A = -0.1875\ F \cdot l$ and
$M_B = +0.1563\ F \cdot l$

Given the full dependency of the resistances at A and B, the first plastic moment must occur at A and so the limit state function for exceeding the elastic stage is given by

$G = M_p - 0.1875\ F \cdot l$

A calculation according to chapter 4.33 leads to:

$\mu(G) = 300 - 0.1875 \cdot 100 \cdot 10 = 112.5$

$\sigma(G) = \sqrt{(30)^2 + (0.1875 \cdot 20 \cdot 10)^2} = 48.02$

$\beta = 112.5/48.02 = 2.34$
$P_f = 9.575 \cdot 10^{-3}$

In case the beam at A has *brittle properties* (e.g., because the beam is fixed to the rest of the structure using bolts that have no ductility), this must be considered as the system failure probability. And, of course, unrelated strengths or weaknesses might appear at any of the sections.

However, if the beam has *ductile properties* an increase of F is possible with cross section at A acting as a plastic hinge and at B with still increasing load by a still increasing bending moment. The final load bearing capacity is reached only if also at B the bending moment capacity is reached.

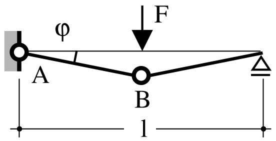


Fig. 4/21: Failure mechanism ...

Let us follow the upper bound theorem of the Plasticity Theory. The structure will collapse in a beam mechanism with rotation φ at hinge A and rotation of 2·φ at point B. So the work done at the plastic hinges is given by:

$W_{int} = M_{PA} \cdot \varphi + 2 \cdot M_{PB} \cdot \varphi$

The displacement at the force is $0.5 \cdot l \cdot \varphi$ and so the work done by the load:

$W_{ext} = 0.5 \cdot F \cdot l \cdot \varphi$

From this, with $\varphi = 1$, the limit state function for the system collapse is:

$G = M_{PA} + 2 \cdot M_{PB} - 5 \cdot F$

In case of full dependency of M_{PA} and M_{PB} this leads to:

$G = 3 \cdot M_P - 5 \cdot F$

The resulting failure probability calculated as above is:

$P_f = 1.474\ 10^{-3}$

If the two bending plastic moments at A and B are for instance fully independent (not a very reasonable assumption for two cross sections in one beam, but only for the sake of discussion), then the analysis becomes more complicated. One cannot be sure that the first hinge will develop at A, as it also may show up at B. The two events do no longer exclude each other. So the event of the end of the elastic stage is given by:

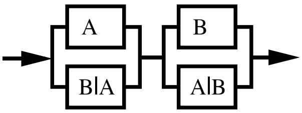


Fig. 4/22: ...and its presentation

$M_{PA} > 1.875 \cdot F$ or by $M_{PB} > 1.562 \cdot F$

For these two events the result is $P_{fA} = 9.575\ 10^{-3}$ (as before) and $P_{fB} = 0.455\ 10^{-3}$. So the conclusion that the probability of first yielding is bounded by:

$$9.575 \cdot 10^{-3} \leq \text{P(first yield)} \leq 9.575 \cdot 10^{-3} + 0.455 \cdot 10^{-3} = 10.03 \cdot 10^{-3}$$

is correct. And even with full independency of the plastic moments at A and B the increase of the probability of first yield is very small.

Once a plastic hinge at one of the two critical locations has formed, and assuming ductile material, the load may further be raised till the second location also becomes plastic. Fig 4/22 gives a schematic representation. Whatever failure path was followed, in the end the beam mechanism already discussed before and the limit state function for system collapse is still given by:

$$G = M_{PA} + 2 \cdot M_{PB} - 0.5 \cdot F \cdot l$$

However, now M_{PA} and M_{PB} are independent and the failure probability follows from:

$$\mu(G) = 300 + 2 \cdot 300 - 5 \cdot 100 = 400$$

$$\sigma(G) = \sqrt{(30)^2 + (2 \cdot 30)^2 + (5 \cdot 20)^2} = 120$$

$$\beta = 400/120 = 3.32$$

$$P_f = 0.447 \cdot 10^{-3}$$

It is worth to notice the differences between the two cases.

c) A portal frame

As a second example consider the portal frame shown in fig. 4/23. The frame is loaded by a vertical load F and a horizontal load H. The elastic bending moment diagrams resulting from some combination of these two loads has also been drawn.

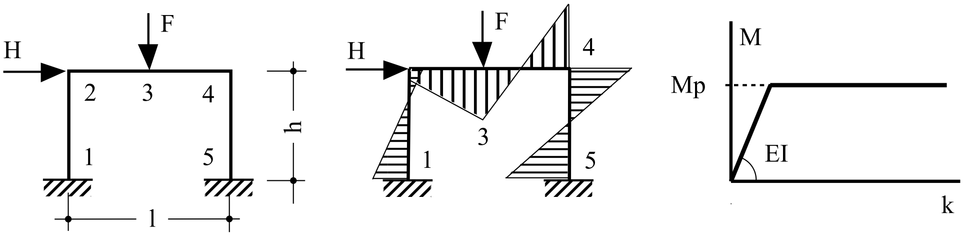


Fig. 4/23: Statical system of a frame, elastic bending moment diagram and beam behaviour

In a similar way as for the simple beam, it is stated that the structure has 5 potentially critical cross sections where yielding may occur.

The probability of failure of each of these elements of the system may then be written as:

$$P_{fi} = P(G_i = M_{p_i} - M_i < 0) \quad (i = 1, 2, \ldots, 5)$$

where M_{pi} is the full plastic moment capacity of the cross section i and M_i the corresponding bending moment resulting from the theory of elasticity. The event that at least one of the 5 cross sections has insufficient capacity, is a series system with 5 elements. Based on eqn. (4.50), the system failure probability lies within the following two bounds:

$$\max[P_{f_i}] \leq P_f \leq \sum_{i=1}^{5} p_{fi} \qquad (i = 1, 2, \ldots, 5)$$

In the case of pure brittle material behaviour, again, this may be close to a real collapse of the structure. Given, however, some degree of rotation capacity, plastic hinges may occur until a full failure mechanism has been developed. In this case there is not just one possible mechanism, but there are three of them as indicated in fig. 4/24, namely a beam mechanism, a sway mechanism and a combined one.

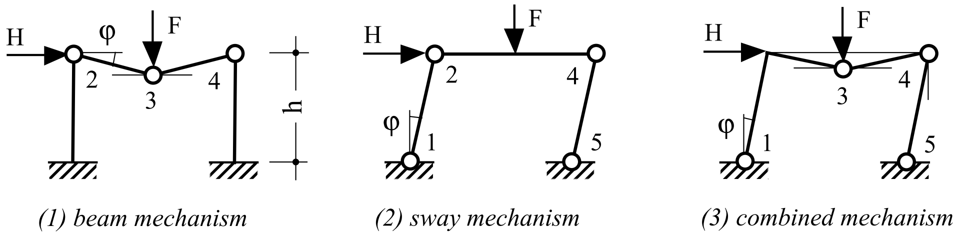


Fig. 4/24: Failure mechanisms

The task is now to formulate the corresponding limit state functions. The principle is the same as in the single beam example. The results for the beam, the sway and the combined mechanism, respectively, are:

$G_1 = (M_{p2} + 2 \cdot M_{p3} + M_{p4}) \cdot \varphi - F \cdot 0.5 \cdot \varphi \cdot l$

$G_2 = (M_{p1} + M_{p2} + M_{p4} + M_{p5}) \cdot \varphi - H \cdot \varphi \cdot h$

$G_3 = (M_{p1} + 2 \cdot M_{p3} + 2 \cdot M_{p4} + M_{p5}) \cdot \varphi - H \cdot \varphi \cdot h - F \cdot 0.5 \cdot \varphi \cdot l$

Failure of the structure will happen if the loads are large enough to provoke just one of the three mechanisms. So, the three mechanisms together form again a series system. The system level probability of plastic collapse is:

$$\max[P_{fi}] \leq P_f \leq \sum p_{fi}$$

where $P_{fi} = P(G_i < 0)$ with $i = 1, 2, 3$.

In summarising: due to the ductile properties of the structure there is no structural failure after the exceedance of the load bearing capacity of just one „element" (critical cross section). Just three or four cross sections need to reach their yield limit until the system collapses. So, each single mechanism is in fact a parallel system. However, the ductile parallel system differs physically and therefore also in its mathematical elaboration from the parallel system treated in 4.72. All mechanisms together form a series system.

In order to treat also this example numerically let's set the units kN and m as before and $l = 10$, $h = 5$, and $\varphi = 1$. The beam plastic moment is set to $M_{p3} = N(250;20)$ and all column moments to $M_{pi} = M_p = N(80;7)$. This leads to the following set of limit state functions:

$G_1 = 2 \cdot M_p + 2 \cdot M_{p3} - 5 \cdot F$

$G_2 = 4 \cdot M_p - 5 \cdot H$

$G_3 = 4 \cdot M_p + 2 \cdot M_{p3} - 5 \cdot H - 5 \cdot F$

Introducing the loads with F = N(50;15) and H = N(20;8) the calculation of the respective reliability indices and failure probabilities is easy. Using a FORM analysis (see 4.42), for each mechanism the resulting reliability indices for the three failure mechanism result in 4.76, 4.51, and 4.79, respectively. The elementary simple lower and upper bounds for the system failure probability can now be calculated as:

$$3.3 \cdot 10^{-6} \leq P_f \leq 5.1 \cdot 10^{-6}$$

The difference between the upper and lower bound is very small. The point is that the combined mechanism is dominating the reliability. The other two mechanisms have a much lower failure probability. When a number of mechanisms have similar failure probabilities the bounds may be wider and it may be worth to use more advanced methods and take care of correlation between the mechanisms. Alternatively, also a full Monte Carlo analysis may be done.

5. Assessment, Judgement and Quality Assurance

5.1 Risk versus Safety

The notion “Safety” describes the state of a system operating with an acceptably small level of risk. Risk is, if necessary, reduced to values below this acceptable level by appropriate safety measures, which often are quite costly. Since long it's known that absolute safety cannot be achieved.

Establishing levels of acceptable risk is a matter for society at large and, thus, can have a significant political dimension. However, in the absence of political direction, it is the public authorities, representing society, who attempt to fix the observable criteria in an absolute manner. For risks judged to be below some acceptance level, and quite generally for risks where no such level is defined, authorities intervene, e.g., by requiring industry to implement ever more costly safety measures.

One thing, however, is clear: “... if our priorities in managing risks are not cost-effective, we are, in effect, killing people whose premature deaths could be prevented ...” (*Okrent*, *1980)*. Thus, in the end, keeping the risk for life and limb of people below acceptable limits in a cost-effective way is of prime concern.

5.11 Risk is a multifaceted concept

a) Randomness in risk analysis

In technical contexts, the term risk is generally understood as a function of the consequences of a possible event and of the occurrence frequency of such an event. The simplest function for relating the corresponding values is the product of these quantities. In fig. 5/1, therefore, risk is shown as a rectangle, defined by the two quantities.

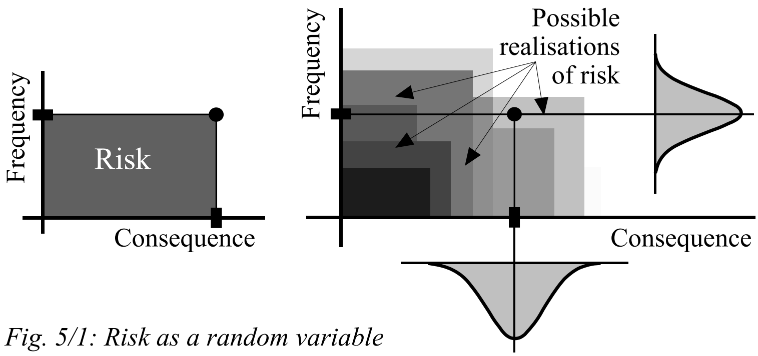


Fig. 5/1: Risk as a random variable

Admittedly, neither the consequences nor the frequency of an event can be known accurately in advance. Both are random variables, which, particularly for infrequent events, are poorly based on statistics. This uncertainty is represented in Fig. 5/1 by the probability density functions (pdf’s) for frequencies and consequences. Naturally, if the input values are randomly defined, then the product of both variables is a random quantity as well, described by its expectation and its variance. This should be recognised.

In those situations one may consider the risk as a random variable and when comparing risks take the expectations. Another option is to define the risk directly as the expectation of the consequences, taking all uncertainties on board. To some extent it is just a matter of wording.

Of course, all of the numbers used in defining risks reflect the opinion of those involved in doing the analysis. This is nicely shown in *Holicky & Schneider, 2001,* and *Bradbury & Schneider, 2001*. Both papers show – in the sense of benchmark studies – the large influence and scatter of the opinions of about 10 well known experts on judging well-defined problems.

b) Scaling frequency of hazardous events

The frequency of a hazardous event is a matter of probability per time, per month, year or centuries. The scale normally is chosen quasi-logarithmic and may be fixed by respective numbers. Often linguistic scales are used, e.g., “extremely rare – very rare – rare – not rare – frequent” or “once per 100 years – once per 10 years – once per year – once per month – once or more per week”, or by any other suitable linguistic or numerical scale.

c) Defining and scaling consequences of hazardous events

Depending on the problem at hand there is a multitude of possible consequences to be observed. The following items are just examples for what might be to consider:

- Life and limb of people (in most cases reduced to the number of people killed by an event), also addressed as fatalities,
- Amount of damage to property (in monetary units),
- Breakdown of electricity or water supply for more than, e.g., an hour, a day, etc.,
- Unavailability of a technical facility,
- Unavailability of transport systems, e.g., caused by flooding, strike, etc.,
- Area of contaminated ground (e.g., in km^2),
- Damage to the public image of a firm, a school, a hospital, etc.

See in this respect, e.g., *BAFU, 2008*.

Also the consequences of events are often fixed by numbers on a quasi-logarithmic scale. Where this is not feasible, and from a communication point of view, also verbal scales may be helpful, e.g., “non or small – average – big – very big – catastrophic” or any other expressions better defining the respective consequences.

d) Risk aversion

Experience shows that, e.g., one person killed in a traffic accident per day is quite easily accepted while two or three accidents per year killing more than 100 people each are absolutely unacceptable. Though the number of people killed and the accumulated risk over the days, weeks, and months is the same, the perception of these risks is different.

This distorted perception of (so-called) objective risks may be modelled by:

$$R_{perc} = R_{obj} \cdot A(E) \qquad \text{or, alternatively, by} \tag{5.1}$$

$$R_{perc} = p_E \cdot E(S)^a \tag{5.2}$$

wherein R_{perc} the perceived risk, R_{obj} the (so-called) objective risk, p_E the probability of an event, E(S) the expectation of its consequences and the exponent $a > 1$. The models and both, the factor A(E) and the exponent a are of clearly subjective nature and may be evaluated in discussion with experts *(Schneider, Th., 1981)*.

Risk aversion is not only present when discussing human safety, but also in relation to monetary losses. Actually it is the driving force behind the world of the insurance: people prefer a small sure loss above a possible large one, even if the objective expectation of the large loss is smaller. The type of modelling is closely related to the famous notion of "utility" (see *Von Neuman, 1944).*

Risk aversion is observed and is a fact. However, discussing at the same time two different numbers measuring the risk allocated to the same situation is often disturbing and very often results in misunderstandings. One should either have the objective risk or the perceived risk on the desk for discussion.

Engineers in general like the straightforward objective risks, but considering perceived risks is sometimes unavoidable as being a part of (political, emotional) reality. One should also have in mind that the so-called objective risk only leads to correct results when "the game" can be played an infinitive number of times. This is of course not always in a sufficient way the case in reality.

5.12 Risks to life and limb of persons

A distinction is made between risks to persons and risks to property. Risks to persons are often dominant – and not simply because of the cost of a damaging event, but also for ethical and legal reasons. Generally, risks to persons are measured by the probability of death. This is because fatalities are easily counted, unlike injuries with their varying degrees of severity. Furthermore, the number of deaths often exhibits a more or less constant ratio to the number of injured. Therefore, risk of death is often used as representing the total risk with respect to life and limb.

Mean death probability per year and 100'000 persons	
All included:	
50	25 years old
80	35 years old
200	45 years old
600	55 years old
1200	65 years old
3000	75 years old
Occupational death probabilities:	
100	Lumber jack, timber transport
90	Forestry work
50	Worker construction site
15	Chemical industry
10	Mechanical industry
5	Office work
Different death probabilities:	
400	Smokers: 20 cigarettes a day
300	Drinkers: 1 bottle of wine a day
150	Drivers: Sports motor cycling
100	Flyers: Delta flying as hobby
20	Car drivers (20–24 years old)
10	Pedestrians, household workers
10	10,000 km/year car travellers
5	Hickers in the mountains
3	10,000 km/year motorway drivers
1	Flyers: Plane crash per flight
1	Living in buildings: death in fire
1	10,000 km/year train travelling
0.2	Death in earthquake (California)
0.1	Death of being struck by lightning

Fig. 5/2: Mean death probabilities

When evaluating risks, a differentiation is made between individual and collective risk. The former is related to the risk to which an individual person is subjected in a particular situation. An individual clearly orients himself with respect to this quantity and, of course, his or her preferences and lifestyle. In contrast, collective risk includes all persons who are subject to hazards in a particular situation. Collective risk is clearly a major concern for the operator of a technical facility (e.g., the operation of a railway) or of a process (e.g., in a chemical plant), as well as for society in general.

Some facts: In Switzerland, a country with some 8 million inhabitants, around 20'000 fires occur every year, in which, according to long-term statistical records, about 5 persons per million inhabitants die. Thus the individual probability of death due to fire in Switzerland is – taking the average including the neighbouring countries in central Europe – in the order of 1 per 100'000 persons and year. This probability may be compared to other probabilities, e.g., the roughly 10 times greater probability of dying in a traffic accident, or the approximately 100 times greater occupational risk of a forester, or the 1000 times greater probability of dying as a 60-year-old.

Fig. 5/2 presents, in round numbers, average individual death probabilities that apply more or less for Switzerland.

A wealth of further data of this type can be found in *Fritzsche*, *1992* and *Tengs et al*, *1995*. It is obvious that age-dependent probabilities dominate.

Added to this quasi-natural risk are the quite different occupational risks and those risks related to individual activities of various types, and to personal lifestyle. With good approximation, specific numbers may be added to determine the total probability of death for a given individual.

Traffic-related risks are of particular interest for comparison purposes; in this table they are standardized for an average yearly travel distance of 10'000 km.

Some of the risks listed in fig. 5/2 would probably be given a higher value, if someone were simply asked to give his/her subjective estimate, e.g., the probability of dying in California in an earthquake. This reflects the term risk aversion discussed earlier.

5.13 Risk acceptance

Numerically fixing acceptable individual risk to persons largely depends on a) how voluntarily an individual person undertakes a given activity, and on b) the capacity and possibility of that person to effect a reduction of the risk by appropriate personal behaviour *(Schneider, 1994)*. Fig. 5/3 brings these aspects together, showing some typical activities. Fig. 5/4 presents fatality risks per 100'000 persons per year which generally might be found reasonable and acceptable.

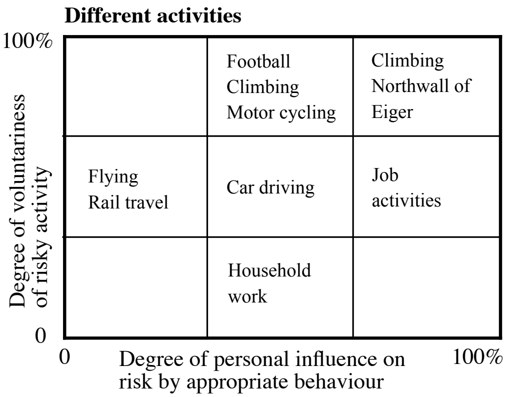


Fig. 5/3: Different activities ...

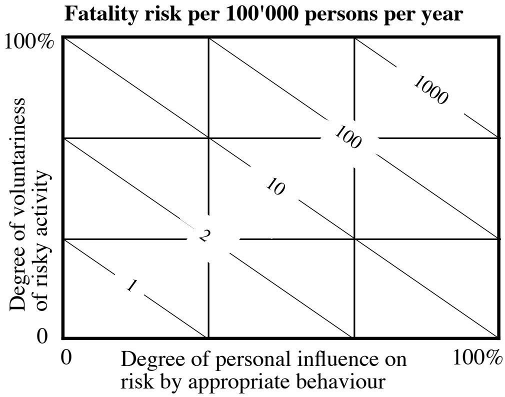


Fig. 5/4: ... and associated fatality risks

When it comes to the protection of residential areas from, e.g., industrial activities it seems that in various countries a broad societal consensus has been reached to fix an acceptable individual risk according to the bottom left corner of fig. 5/4. If this value were adopted, it would fix the acceptable fatality risk of an uninvolved individual to, say, an annual risk of 1 in 100'000, i.e., an individual risk of 10^{-5}/year.

For industrial activities, giving profits to one group and burden involuntary risk to others, often a value of 10^{-6}/year is taken as guidance.

From society's point of view, however, individual risk is usually not of primary concern; rather, it is the frequency and severity of events damaging life and limb of people in, e.g., an individual fire in Switzerland resulting in 5 or more fatalities.

5.14 Communicating and discussing risks

Communication between experts in the field and the community at large is difficult. Reasons are lack of necessary knowledge, limits of understanding, mutual distrust, and often arrogance of those involved. There are, of course, different opinions, as often at the side of, e.g., a chemical plant are the experts, while at the other side is the public, which normally suffers from risks pertaining to the plant. And, of course, risk aversion plays a significant role.

Communication of risks in order to finally reach a consensus is a matter of a calm and patient discussion between all those involved in a case, e.g., the placement of a storage plant for radio-active waste. And it needs easy to understand visual aids. The following sections foster such search.

5.15 F/N-Diagrams

Risks are often visualized in a frequency versus consequences (F/N) diagram. It is important to note that such diagrams are valid only for a specific case, activity, facility, region, or country.

Both axes in F/N-diagrams are defined on a continuous logarithmic scale (fig. 5/5). The frequency F is normally defined per year, while the consequences N may be seen in quite different indicators. Of course, damage to life and limb of people may be seen as the most important consequences as shown in fig. 5/5.

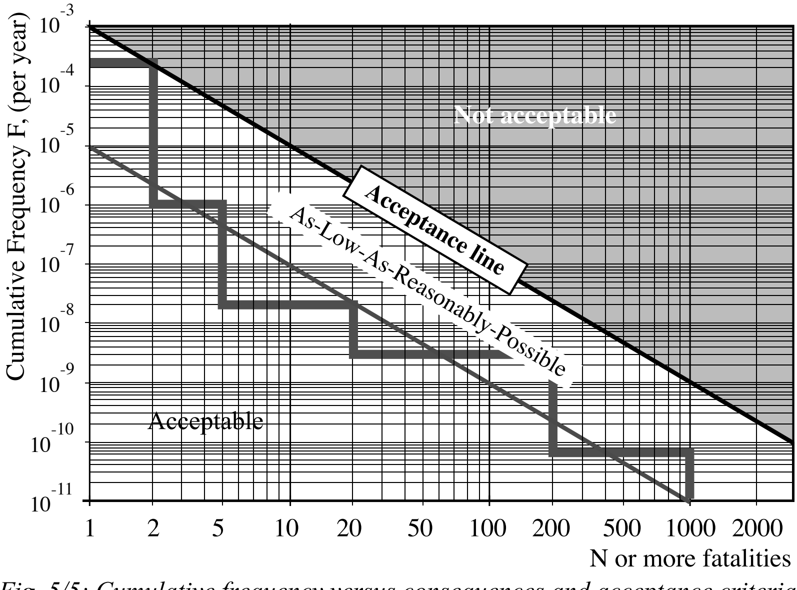


Fig. 5/5: Cumulative frequency versus consequences and acceptance criteria

An acceptance line arbitrarily or after having reached a consensus defines risks, which the authority issuing the diagram judges to be acceptable. This means that such diagrams are valid only for a specific case, activity, facility, region or country.

It might seem logical if the acceptance line took the form of a neat diagonal, i.e., showing that frequency should decrease by one decimal as consequences increase by the same factor, thus keeping the risk at the same level. In fig. 5/5, the steeper acceptance line reflects a disproportionate decrease in accepted risk. In other words, it points to some rather high risk aversion, defined here by an aversion exponent of 2 (see section 5.11, d)). For even more objections in this context see, e.g., *Evans & Verlander, 1997.*

About two orders of magnitude lower than the acceptance line, a so-called line of insignificance is often drawn, separating the region below left, whose risks can justifiably be classified as insignificant. The space between these two lines is called the ALARP region (“**A**s **L**ow **A**s **R**easonably **P**ossible”). Risks falling in this region should be managed by applying safety measures, provided this is technically, operationally, and economically possible and practicable.

Risks situated below the acceptance line may be considered acceptable, but there may be some disagreement between those instances, e.g., industrial enterprises, introducing the risks and those groups of people affected by the risk, as to whether a further risk reduction should be introduced by implementing additional measures. The ALARP principle can be a good guide in such discussions.

In fig. 5/5 the amount of damage caused by a possible event is represented by the number of fatalities ascribed to this event. In frequency-consequence diagrams, however, a variety of other damage indicators (flooding land, affecting life and limb of animals, damaging cultural property, contaminating soil or surface or groundwater, etc.) may be considered (see section 5.11 c)).

When the risks of a specific project or operation are assessed, the results of the analysis may be condensed to a curve or a stepped line in such a diagram. When performing such an analysis for each conceivable damaging event – engineers prefer the term “hazard scenario” (see section 1.34) – the damage, i.e., the number of fatalities, and the respective probability of occurrence, need to be assessed. These numbers define the risk associated with the hazard scenario under consideration.

As an example: From five earthquake hazard scenarios defined by different severity and frequency, the five steps shown in fig. 5/5 are obtained. The most severe scenario with 1000 people killed and associated with the smallest frequency forms the basis. The next scenario with 200 people killed but higher frequency is put on top, etc. Adding up all respective rectangles yields the total risk potential of the earthquake hazard to the problem under discussion.

By definition, the risk associated with a project, defined by the stepped line, may not exceed the agreed acceptance line – irrespective of who defined it and where it may lie. If risk does exceed accepted levels, measures must be implemented to bring the risk below the acceptance line.

5.16 F/C-Matrix

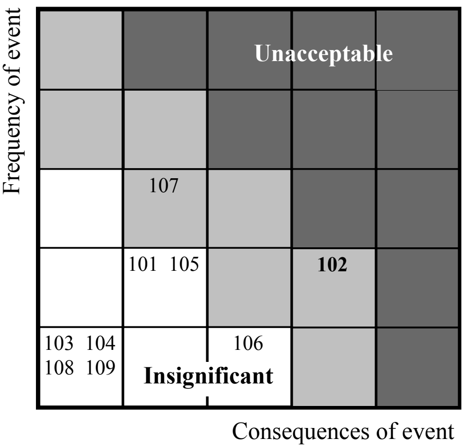


Fig. 5/6: Frequency-Consequences matrix

As most of the background information on frequency and damage of a hazard scenario is based on personal judgement, one should not be fanatic about very exact numbers. In many practical cases it is good enough or even more appropriate to depict information about events and respective activities on an F/C-Matrix.

It is suggested that such a matrix is divided into 25 sub-squares as shown in fig. 5/6. Experience shows that dividing into 9 or 16 sub-squares in the end results in hazard allocating difficulties. The lower left sub-square hosts the events with the least consequences and the smallest frequencies, the upper right square the most dangerous ones.

How to index the consequences of an event and its frequencies was discussed in section 5.11 a).

The use of quasi-logarithmic scales on both axis are suggested. In that case events of equal risk are on a line of slope -1.

Each event or hazard scenario affecting the problem under discussion is described in a report and is given a number, e.g., 107. These numbers are inscribed in the appropriate box. The position of the numbers indicate what should be done with respect to risk reduction. Scenarios in the lower left corner seem to be insignificant and do not need further attention. A scenario allocated in the upper right corner is judged inacceptable and clearly asks for attention and action to move it to any better place. Scenarios in the lighter shaded grey should be handled as ALARP, described in the previous sections.

Be aware of misinterpretation and dangers:

- a few light grey blocks together may represent a larger risk than one dark grey block. This gives the opportunity for manipulating.
- the FC matrix is not telling you what is acceptable and what is not: it's just a way of presenting and comparing risks of different kind.

See also the critical review in *Cox, 2008*.

Clearly, the F/C-Matrix is just the place to guide attention, to serve communication and to manage necessary action. This is its only purpose. The base of putting the numbers of events into the square is the list of the events, resp. hazard scenarios analysed. Preparing this list and in allocating the numbers to the sub-squares is not an easy task and needs intensive discussions between all people involved.

And, of course, in case one is interested in different kinds of consequences, e.g., in risk to life and limb, in risks to property measurable in monetary units, or in the availability of a facility, or in the image of a company, for each of these properties, such an F/C-Matrix must be drawn.

5.2 Life-saving efficiency of safety measures

5.21 Introduction

Each measure M applied to reduce a given risk costs money. The ratio of the safety cost SC_M of the measure to the associated risk reduction ΔR_M due to this measure is a number expressing monetary unit per damage unit, e.g., € 300'000 per human life saved. This ratio is termed life-saving cost LSC_M of this measure *(Stiefel & Schneider, 1985)*. The smaller this number, the more efficient is the safety measure.

The life-saving cost is an objective quantity, but it is certainly not to be construed as defining the value of an individual human life.

Such life-saving costs can be easily determined. For example, at a particularly dangerous road junction an average of two people die per year as a result of traffic accidents. A complete redevelopment of the junction might reduce fatalities to almost zero, but would cost € 8 million. Using normal discounting procedures, this sum might translate into annual costs of 800'000 €. A closer investigation might reveal that 75% of the redevelopment funds should be allocated to reducing the risk while the rest might be charged to increase road comfort, reduction of noise, etc. This results in the following simple calculation:

LSC_M = (0.75 x 800'000 €) / 2 lives saved = 300'000 € / human life saved

If redevelopment of the junction is undertaken, then – implicitly – it expresses the consensus that the community is prepared to spend € 300'000 to prevent a fatal traffic accident at that location.

5.22 Life-saving costs invested in structural safety

Quite central in this context might be the question how much money is normally invested into structural safety, what the life-saving costs invested into structures are. Of course, what follows is based on a number of assumptions and, thus, results are just rough estimates (*Schneider, 2000).*

In order to get a somehow valid answer to this question let us have a look into section 4.33 and fig. 4/5. The problem at hand is described by the limit state function

G = R – S

Fig. 5/7 shows the probability density function of the safety margin in greater detail. Highlighted in the figure is the failure probability related to one standard deviation, e.g., $p_f = 0.16$.

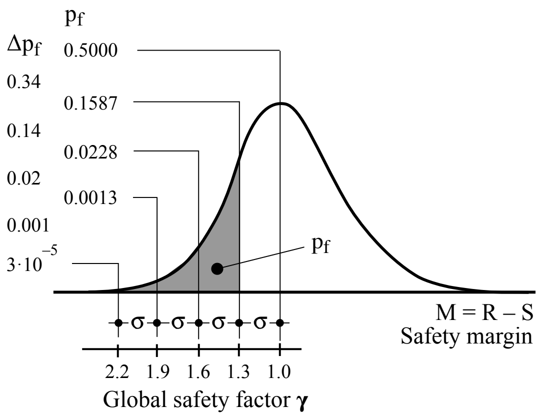


Fig. 5/7: Probability density of the safety margin

Assume that an overall safety factor of 2.2 was used when dimensioning the structures (though dimensioning rules, codes, etc., during the last 150 or so years are manifold), and that this factor is translated into four-times the standard deviation σ of the safety margin, resp. into a failure probability of roughly $p_f = 3 \cdot 10^{-5}$ or $\beta = 4.0$, all assuming that the safety margin is normally distributed.

Now, here is a question: how much money is invested in a structure in order to make it safe. Here is a way: The value of the building stock (houses, schools, office buildings, factories, etc.) in Switzerland is estimated to some $2'500 \cdot 10^9$ €. From this about 30% may be buried in respective structures, i.e., $750 \cdot 10^9$ €. Not all money invested in a structure, however, is related to make it safe. A guess is that the safety part amounts to some $200 \cdot 10^9$ €.

About 8 Mio. people actually live in Switzerland. The number of people present in buildings of any kind at any time might be about 6 Mio. In case, by some hypothetic reason, all structures would fail all-of-a-sudden the guess is that some 3 Mio. people would be injured. For some one Million people this failure would possibly be a matter of life or death.

If a safety factor of 1.0 had been used, 50% percent of these people would have been killed, i.e., 500'000 people, while raising the factor from 1.0 to 2.2, only $3 \cdot 10^{-5} \cdot 10^6 = 30$ people would die, while 499'970 would survive.

The conclusion: the investment of $200 \cdot 10^9$ € into the safety of all building structures in Switzerland saves almost 500'000 people, or, in other words: 400'000 € are invested into the safety of the structures he or she is using, per person. In the wording of life-saving cost, this is:

LSC = $200 \cdot 10^9$/500'000 = 400'000 €/human life saved

Even more interesting is a look to the development of the life-saving cost with respect to an increasing safety factor. Let us assume that to each of the steps of 0.3 of the safety factor $50 \cdot 10^9$ € are allocated. An increase of the safety factor from 1.0 to 1.3 is cheap. It reduces the failure

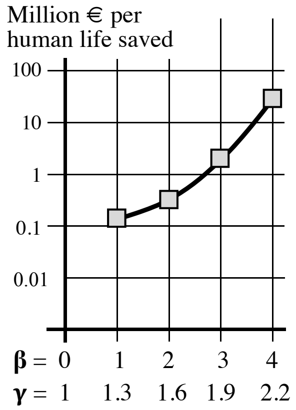


Fig. 5/8: Life-saving cost versus safety factor

probability from 0.5 to 0.158, i.e., by a difference of 0.34. This translates to $50 \cdot 10^9 / 0.34 \cdot 1 \cdot 10^6$ = 147'000 € per additional person saved.

The next step from 1.3 to 1.6 saves less additional people and saving their lives costs 368'000 €/person. The next two steps from 1.6 to 1.9 and to 2.2 are even more expensive and cost 2'300'000 and 39'000'000 €, resp., per additional person saved. Fig. 5.8 shows this exorbitant increase on a logarithmic scale.

Don't take the numbers as serious as they are shown here. There are lots of guesses and assumptions behind the above derivation. Nevertheless, what is stated here gives an idea of what might be invested into structural safety.

New concepts like, e.g., the Life Quality Index (LQI) and research on the statistical value of life might be of interest. Answers to this question may be found in *Nathwani et al., 1997, Rackwitz, 2004; Kübler & Faber, 2005; Panday et al., 2006; Nathwani et al., 2009; Fischer et al., 2011; Lind & Nathwani, 2012; Fischer et al., 2013; Faber & Maes, 2010; Faber & Virgues-Rodriguez, 2011)*

5.23 Other areas

Life-saving costs € per life saved	
100	Multiple vaccine 3rd World
$1 \cdot 10^3$	
$2 \cdot 10^3$	Installation of x-ray equipment
$5 \cdot 10^3$	Wearing motor cycle helmet
$10 \cdot 10^3$	Providing cardio-equipped ambulance
$20 \cdot 10^3$	Tuberculosis checks
$50 \cdot 10^3$	Providing helicopter for emergencies
$100 \cdot 10^3$	Safety belts in cars
to	Reconstructing road junctions Providing kidney dialysis units
$500 \cdot 10^3$	Building structures
$1 \cdot 10^6$	
$2 \cdot 10^6$	
$5 \cdot 10^6$	Zurich fast rail system, AlpTransit
$10 \cdot 10^6$	Swiss earthquake code
$20 \cdot 10^6$	Safety measures in mines USA
$50 \cdot 10^6$	
$100 \cdot 10^6$	Tall Buildings regulations in GB

Fig. 5/9: Life-saving costs spent in different areas of activity

Life-saving costs as derived above for building structures can be compared with costs observed in other areas. The table in fig. 5/9 shows examples for the order of magnitude of life-saving costs with respect to different measures. The large spread in the numerical values is surprising.

The obvious main question seems to be: "What level of life-saving costs is acceptable or reasonable?" This, of course, is a matter of judgement based on both economical and ethical principles and should be discussed within society. To some extent the principles should be the same for all kinds of natural or technological threads.. Looking to actual practice, it is obvious that society is far from having achieved a consensus. Nonetheless, given restricted financial resources, it is optimal to invest money first in safety measures associated with low life-saving costs. Thus, for the same total expenditure on safety, more lives can be saved.

During the construction of Zurich's fast rail system in the late eighties', little time was spent considering measures with life-saving costs under € 3 million per life saved. Measures with life-saving costs above € 10 million were ignored. Quite similar numbers have been adopted to assess safety measures related to the large railway project tunnelling the Swiss Alps (AlpTransit) in the nineties', and in similar projects in Europe. Nowadays, 5 to 10 million € seem to be standard.

In looking and reflecting on all of the above and referring to Figures 5/3 and 5/4 one could argue that life-saving costs also are a matter of the degrees of personal influence on risk by appropriate behaviour and of the voluntariness of the risk allocated to the character of the activity.

Thus, in order to save a keen mountain climber from a delicate situation in the alps 10'000 € might be appropriate and enough while 10 Million might be well invested to save a person who is charged to perform a societal meaningful difficult task in a dangerous place. By the way: €, CHF, US$, and £ in 2016, in view of all other uncertainties contained in these numbers, are almost the same.

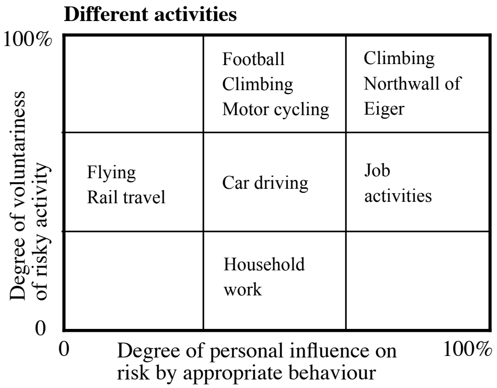


Fig. 5/3: Different activities ...

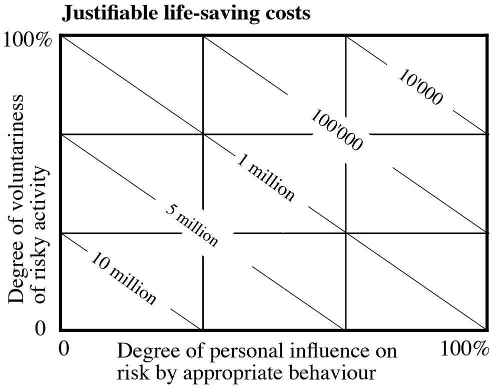


Fig. 5/10: ... and justifiable life-saving cost

The question might arise as to which areas of activity the above considerations apply. Theoretically, the answer is easy: everywhere. For if rules are set, then they should be the same for all areas of application.

These include, for instance, also the very costly medical treatment of patients during their last years and months of life. In fact, also here one could argue about whether the cost of one more month of – possibly quite delicate – additional lifetime is acceptable while with this amount of money the life of many children could be saved.

Many more arguments and examples on such applications can be found in *Pandey et al., 2006* and in *Tengs et al., 1995*.

5.3 Target reliabilities for structures

Under the assumption that *gross human errors are absent* from the problem under consideration, the target probability of failure, theoretically at least, could be obtained from an optimisation of overall costs, including the costs of failure as expressed in eqn. (1.3) in section 1.13. The different terms indicated, however, are not easily accessible, and the results of an analysis of the probability of failure are dependent on assumptions made. Therefore, generally, it is wise to take another approach to answering the question *"How safe is safe enough?"*.

5.31 Calibration to existing practice

The key word to fixing target reliability levels is *calibration to existing practice*, assuming that existing practice is optimal. There is some reason to believe that existing practice *is* optimal because practice would change quickly if designs of structures often resulted in failure. On the other hand, design according to existing practice may, unnoticed, lead to results that are too far on the safe side and not optimal at all, and only departure from existing practice would reveal the size of such hidden margins.

Existing practice is partly established in codes and standards. Thus, calibration should be based on existing codes. There is a problem, however, in that the stochastic parameters of the variables and the uncertainties of the design models of a code are generally not known. A way to proceed is to design a set of typical structures or structural elements according to existing codes. Further, the stochastic parameters of the variables and the model uncertainties should be tentatively fixed.

From these assumptions, the reliability index β of each of the elements with respect to their requirements can then be evaluated. Of course, each of the items will have a different β and, therefore, several iterations of calibration calculations are necessary in order to iteratively change back and forth all assumptions until, finally, the reliability indices β for all elements in the set are within acceptable bounds. From within these bounds a target reliability level β_0 would finally be chosen.

A number of such exercises has been undertaken on a national level (see, for instance, *Vrouwenvelder & Siemes, 1987*, and also, for the Eurocodes, *EN, 2007)*. The results are convincing as far as they show that a general reliability level can be fixed. It also becomes obvious, however, that there is no sense in requiring too narrow bounds.

5.32 Reliability differentiation

It is clear that some reliability differentiation is necessary. Reliability requirements with respect to structural serviceability are less stringent than those related to overall structural safety. Reliability requirements on level of member level should depend on the type of failure (brittle or ductile) and the degree of redundancy in the structure (redistribution possible or not).

		Type and use of structure			
		Class 1	Class 2	Class 3	Class 4
Type of failure	Type A	1.5	2.0	2.5	3.0
	Type B	2.0	2.5	3.0	3.5
	Type C	2.5	3.0	3.5	4.0
	Type D	3.0	3.5	4.0	4.5

Fig. 5/11: The idea behind reliability differentiation

In addition, the reference period, one year or the lifetime of the structure, must be considered. Reliability indices concerning the lifetime are smaller, of course, because acceptable failure probabilities per lifetime are – roughly – annual failure probabilities times the expected lifetime of the structure, e.g., 50 or 100 times larger than failure probabilities for one year.

The numbers shown in Fig. 5/11 are quite formalistically chosen but clearly show the idea behind target reliabilities. The numbers may be taken as a rough indication of the order of magnitude for reliability indices β – *on an annual basis.* It should be noted that these numbers are bound to some specific assumptions concerning distribution types: e.g., log-normal for resistances, normal for dead loads, Gumbel for live loads. Also, model uncertainties must duly be taken into consideration.

β values smaller than about 2 may, with some confidence, be translated into probabilities using, e.g., the Standard Normal Distribution tables contained in Annex 7.3 of this book. In the case of larger values one should be more careful. Due to the large number of assumptions and the large degree of epistemic uncertainty, the probabilities seldom correspond to observable frequencies. That is why, in this context, often the term "notional probability" is used. β values bigger than about 2 rather define a more or less big distance to critical states.

With respect to the type and use of structures, the following comments and examples may help to identify applicable reliability indices:

- Class 1: Agricultural structures, glasshouses, etc. Economic consequences of failure low. No hazards to life and limb.
- Class 2: Office buildings, etc. Considerable economic consequences of failure. Low hazards to life and limb.
- Class 3: Bridges, theatres, high rise buildings, etc. Large economic consequences. Medium hazards to life and limb.
- Class 4: Power plants, large dams, etc. Extreme economic consequences. High hazards to life and limb.

With respect to type of failure, the following comments may help to identify applicable reliability indices:

- Type A: Serviceability failure, structure almost in elastic domain
- Type B: Ductile failure of redundant systems with reserve strength
- Type C: Ductile failure, but almost no reserve strength
- Type D: Brittle failure of non-redundant systems

It is obvious that any code provisions legally override the above classifications and the indicated numerical values.

5.33 Target reliability in ISO 2394

To establish the optimal and/or maximum acceptable failure probability for existing or future structures is, in general, not a normal design task. For many reasons national codes and standards prescribe recommendations or even legally mandatory restrictions in this field. The system of safety differentiation as well as numerical values may differ from country to country or even may differ between fields of application within one country.

	Consequences of failure		
Relative cost of safety measure	Class 2	Class 3	Class 4
Large	3.1	3.3	3.7
Medium	3.7	4.2	4.4
Small	4.2	4.4	4.7

Fig. 5/12: Target reliability in ISO 2394

Usually the minimum safety requirements depend primarily or even only on the consequences of failure. Some codes only refer to the human safety aspects and leave the economic optimization considerations completely to the designer. Other codes may include relative or absolute economic losses in the description of a particular consequences class. The motivation may be to have a control over national resources or to guarantee inexperienced buyers of building structures a minimum quality.

In ISO 2394, Annex G (see *ISO 2394, 2015)*, a table with tentative annual target reliabilities for a 50 year design working life is presented. The table stems from the JCSS Probabilistic Model Code issued by the Joint Committee of Structural Safety (see *JCSS, 2001)* and is based on monetary optimisation. The numbers are given as a function of the costs of the risk reduction measure and the consequences in case of failure, both defined relative to the initial construction costs of the structure at hand.

Fig. 5/12 contains the essential parts of this table. It provides, for different structural classes, tentative target reliabilities related to one year reference period and ultimate limit states, based on monetary optimization.

The target reliabilities relate to structural failure events and may thus be used for failure events ranging from component failures to partial or full structural collapse by adjustment of the relative failure consequences. In a similar way the numbers may also be used for both design and assessment by changing the relative cost of the safety measure to achieve the same reliability improvement. The precise data used to find the resulting values in the table may be found in *Rackwitz, 2000*.

Note that in fig. 5/12 the relative cost of safety measures is present which is absent in fig. 5/11 of the previous chapter. The idea is that a higher safety level should be aimed for if safety can be obtained at low cost. To use fig. 5/12 the two criteria of fig. 5/11 (use of building and type of failure) have to be combined first. For instance Class 4 in fig. 5/12 may be associated with the combinations 4C, 4D and 3D of fig. 5/11. One may argue that one needs actually three axes.

The target reliabilities given in figure 5/12 should be seen as indicative for the support of economic optimization. Human life safety also may be incorporated as an economic value for a saved life, often referred to as compensation costs (low) or even better a value as Social WPT (high) following from for instance an LQI type of reasoning. The order of magnitude is 0.5 to 5 Million Euro. However, the value depends on the economic and social circumstances in a country.

It is also important to realise that specified target reliabilities should always be considered in relation to the adopted calculation and probabilistic models and the method of assessment of the degree of reliability. For instance, someone might have the idea to include human errors, though the authors do strongly oppose to this idea and advocate to invest into Quality Assurance measures discussed in section 5.5.

Target reliability levels following from economic considerations, even including a compensation for the loss of life and limb, may be too low from any ethical or social point of view. In principle the economic optimum leads to a specific or average acceptable value. This however may lead to certain persons and activities having a far more than average risk. This may be unethical and lead to additional requirements

Consider for instance a structure like a temporary grand stand, say for a one day event. At first sight it looks that because of the short duration a relatively high risk level would be acceptable. However, a hypothetical person, being present every day per year at another short duration temporary structure (e.g., a workman) still deserves the minimum reliability level. The point is that it is not allowed to take an average between various persons. To some extend it is not unethical to average over various (relatively short) periods for one person, but this might be very difficult to be done correctly. For practical reasons it seems better to have the same maximum acceptable failure rate for every structure and/or activity (see *Faber & Vrouwenvelder, 2014)*. It means for instance that the design wind velocity is the same for a one year as well as for a one day structure (not counting seasonal effects).

5.4 Assessing existing structures

5.41 General considerations

The need to assess the reliability of the structure of an existing building may arise from a number of causes, all of which can be traced back to doubts about the safety or the reliability of a structure (see *Straub & Faber, 2005)*. The fundamental problem then is to answer the question of whether the structure is safe enough. In fact there are only two possible answers: yes, or no!

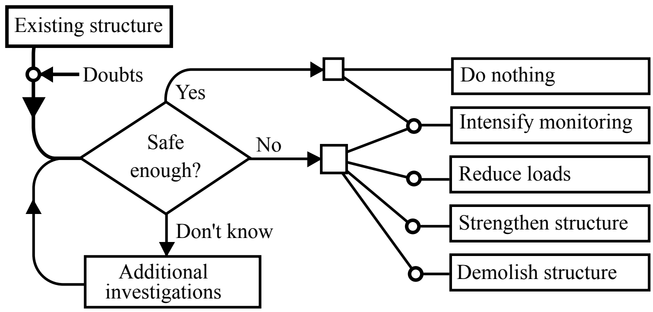


Fig. 5/13: Flow chart used in assessing existing structures

If the answer is *no*, one of the following actions has to be taken: *demolish* the structure and replace it by a safer one, *strengthen* the structure, ask for *reduction of loads* or *intensify monitoring*, all in order to change the *no* to a *yes.* If *yes* is the answer, then to *do nothing additional* and allow continued operation of the structure may clearly be the resulting action, though intensifying the monitoring of the structure is sometimes a good idea. Asking for additional investigations does not circumvent the fact that, finally, a decision is needed. Figure 5/13 illustrates the engineer's situation in the assessment of a structure's safety and points to the key question to be answered.

The assessment of the structural reliability of an existing structure is a difficult task because statements about its possible behaviour under conditions of extreme loading have to be made. Such conditions normally lie outside of the range of experience gained from observing the behaviour under service loads. Also critical for assessing the structural safety is the often rather poor information about the condition of certain structural elements, e.g., with respect to corrosion or fatigue.

The evaluation of the reliability of existing structures should be based on a rational approach. The safety and economy implied by certain decisions are evaluated by means of both structural and reliability analyses taking account of economical considerations. The degree of sophistication depends on the type of structure of concern.

a) Responsibilities

It is clear that the *owner* of a doubtful structure by law is responsible for initiating the safety investigation, since he is liable causally for damage due to the failure of his structure.

The *engineer* is responsible for a careful execution of the investigation and especially for an expert formulation of statements on the safety of the structure and the measures proposed.

The owner is finally responsible for complying with the provisions and measures proposed by the engineer. As a rule the *final decision* de jure is taken by the owner.

b) Basis of assessment

When assessing existing structures it is essential to know for how long the structure is intended to serve its purposes. This period is termed the *residual service life*. It is evident that the assessment of a structure must properly take into account its use during the foreseen residual service life and any particular requirements of the owner. The residual service life is fixed in the *service criteria agreement* discussed under section 1.42.

On the basis of the service criteria agreement and regarding the future use of the examined structure, a list of hazard scenarios likely to act on the structure must be defined and put together in the *Safety plan* discussed under section 1.43. Also the *List of accepted risks* introduced in section 1.44 must be set up.

c) Updating of information

Updating of information about the structure and its present and future use is an important procedure in assessing the reliability of existing structures. Updating is based on *prior information* about the structure and especially collected additional observations and measurements. Pooling all this together results in so-called *posterior information* that serves for assessing the structure.

The additional information comes from:

- Design: the information relevant to this aspect is generally obtained from reports, existing drawings etc.
- Field experience: the experience acquired during operation improves knowledge of the real behaviour of the structure. Data may be obtained from monitoring, inspections, etc.
- Requalification analysis: at this stage, information obtained from both the design documentation and the field experience are critically reviewed and updated and then used to estimate the new conditions of the structure.
- Economic analysis: the potential consequences in terms of direct or indirect costs are evaluated.

More on these points may be found in sections 5.43 and 6.3 and in *Quangwang et al., 2015*.

5.42 Assessment phases

Experience shows that breaking down the assessment of a structure or any other facility into a minimum of two phases is reasonable. Fig. 5/14 presents these phases in a flow chart.

Each of these phases should be complete in itself. It is clear that each should be begun with a precisely formulated contract, usually in written form. The client and the consulting engineer will have to formulate this contract together. Each phase, similarly, ends with the respective report, leaving the owner with his responsibility and freedom of decision.

This freedom is, to be sure, constrained by the recommendations of the engineer and the requirements of the laws governing the owner's responsibilities and the criminal code.

a) Phase I: Preliminary assessment

The purpose of Phase I is to remove existing doubts using fairly simple methods, which must, however, be adequate for the task at hand or – if this is not successful – must allow proposals to be made for subsequent actions. The preliminary evaluation consists of a rough assessment based on inspection, an accompanying study of the available documents, a rough check on the structural safety, and a report.

A detailed inspection of the structure or structural part in question is extremely important, especially the recognition of typical hazard scenarios that could endanger the structure's residual service life. Further, any defects and damage due to excessive loading must be detected. As soon as there is some evidence of danger to humans or the environment, protective measures must be implemented straightaway.

In the case of many existing structures both the service criteria agreement and the safety plan mentioned above will be missing. These documents have to be set up or amended in view of the residual service life aimed at and, as a result, form an important basis for the assessment.

In studying the available documents, an attempt must be made to gain a deep insight into the original situation: which aims were followed, which construction methods and which construction materials were used? How was the economic and organisational climate? Was the work affected by pressure to meet deadlines or low price? These are so-called *quality indicators*.

A study of the static analysis, in addition, provides a wealth of information about codes, calculation, and design methods. At the same time that study also shows where there are reserves of strength which, according to the present state-of-the-art, could be exploited. Based on these documents, plus the drawings, any doubts about a structure's safety can, as a rule, be confirmed or dismissed.

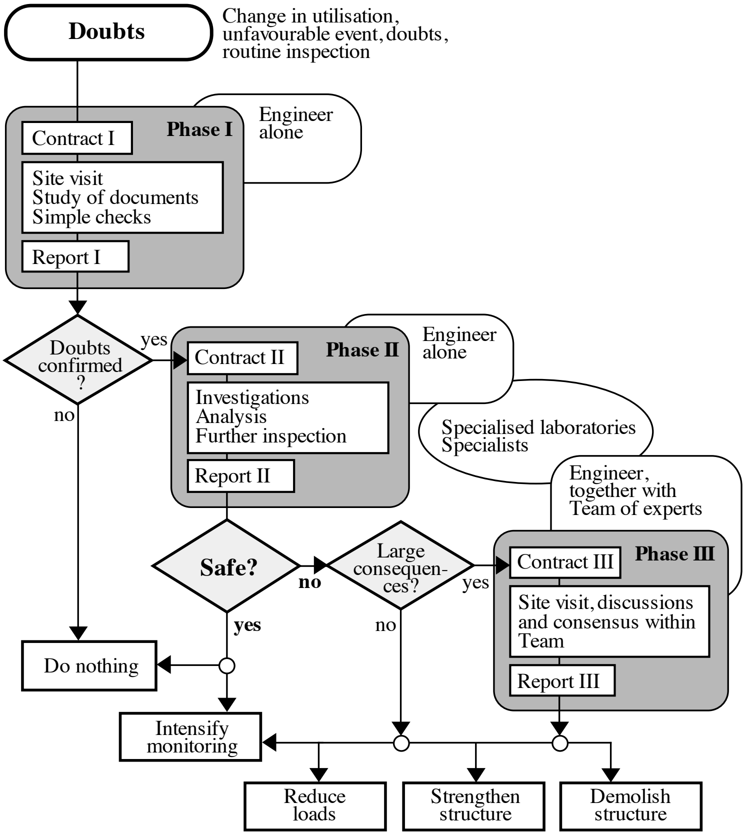


Fig. 5/14: From doubts about the reliability of a structure to remedial action

The information gained in Phase I is summarised in a report for the owner. If the doubts that led to the commission being undertaken cannot be overcome in the course of Phase I, further investigational steps must be undertaken in Phase II.

b) Phase II: Detailed investigations

Structural investigations and updating of information are typical of Phase II. It is only common sense to entrust the same consulting engineer with the work of Phase II, so that the knowledge gained in Phase I is fully utilised. Here, in addition, a specialist firm or agency or individual experts generally have to be called in.

It is sensible and cost-effective to build upon the knowledge gained and the questions remaining from Phase I and to compile a list of points requiring further investigation, thereby specifying what still needs to be checked. The thoroughly prepared investigation should be closely supervised by the consulting engineer. More on this can be found in *Ellis et al., 1995*.

The additional information gained from the investigations can be introduced into confirmatory calculations with the aim of finally dispelling or confirming any doubts as to whether the structure is safe – while remaining aware of the subjective character of this decision. Here, probability methods increasingly find their way into practice.

All results of Phase II are summarised in a report, which again is handed over to the owner. In particular, the report gives information on the structural safety.

If the safety is thought to be inadequate, then intensified monitoring, reduced loads, strengthening, and, if the circumstances justify it, a possible demolition and reconstruction of the structure can be considered.

If the conclusions of this report would result in actions of relatively little consequence, then the investigation can be brought to a close at the end of Phase II. Such would be the case, for instance, when no human lives are endangered and risks of damage to assets can be accepted. Ending the investigation is also acceptable even if one decides upon strengthening, repair, extending the lifetime, or demolition and reconstruction, as long as this decision does not imply inordinate risks or large financial consequences.

For decisions that carry large consequences, the consulting engineer in his report on Phase II should propose proceeding further to Phase III. If the owner is interested in a balanced and unprejudiced assessment, he should be in favour of going ahead with this step.

c) Phase III: Calling a team of experts

For problems with substantial consequences, a team of experts should be called in to check carefully the proposals for the pending decision. The team should comprise, apart from the consulting engineer commissioned to do the work thus far, two or three additional experienced engineers. The owner or the operator is not a member of the team but should supply the team with information as required.

In assessing an existing structure, such a team of experts acts to a certain extent as a substitute for the codes of practice, which for new structures constitute the rules to follow in a well-balanced and safe design. In particular, the acceptance of increased risks should be left to this team of experts. The engineer responsible for Phases I and II of the work should draw the attention of his team's colleagues to all available documents and justify his proposals for the measures to be adopted. The team is well-advised both to inspect the structure and to confer together. The decision of the team should be unanimous and be defended as a team before the owner and, if necessary, publicly. The responsibility for the decision is carried by the team as a whole.

It must be stated, however, that even the opinion of a team of experts is subjective and might be opposed by others. Even teams of experts have been sued by those who felt ill-served by the team's decisions.

5.43 Activating reserves

Rejecting a structure because of insufficient structural safety and, thus, deciding to *demolish* the structure should be considered as a last resort and the least favourable of the options. If *strengthening* is required, this should be carried out with the utmost care so as to cause the least possible disturbance to the existing structure. The best way of solving the safety problem, however, is to *activate* the structure's existing *reserves* of load carrying capacity in order to achieve the required safety level. If this is possible the structure is not disturbed at all.

The main task to be completed in Phase II is to obtain an updated knowledge of the structure. This updating process is related to several problem areas and should be undertaken with all possible objectivity. It concerns mainly the following:

- the basic documents
- the structure as such
- loads and actions
- the material properties
- the structural system
- the methods of analysis and dimensioning.

These questions are discussed in the following sections. Not dealt with, however, is how the structure is to be investigated and what and how data can be collected and updated. Here the reader is referred to the literature (see *CEB, 1989* and *JCSS, 2001)*.

a) Preparing the basic documents

To assess the structural safety of existing structures the *service criteria agreement* and the *safety plan* are of crucial importance (see section 1.4). They have to be updated in relation to the *residual service life*. If not available, e.g., in the case of older structures, they have to be established.

In the service criteria agreement, amongst other statements about the foreseen use of the structure, the expected residual service life is specified. This specification, however, may have to be modified, depending on the results of the investigation. In developing the safety plan attention should be given to those hazard scenarios which are relevant within the residual service life.

b) Assessing the structure

The structure has to be carefully checked for defects, cracks, damage, displacements, deformations, and indications of corrosion, ageing and fatigue. Observed defects must be considered in the assessment of the structural safety. In the case of older structures, crass hidden errors are improbable; they probably would have been detected sooner.

In particular, the relevant structural dimensions have to be checked if the structural drawings are missing, if changes were made to the structure or if there is reason to believe that there are significant deviations from the dimensions given in the drawings. In the re-analysis of the structure the measured values should be introduced, in favourable as well as in unfavourable cases.

Also, the static and kinematic conditions important for the behaviour of the structure (support conditions, movability of supports and joints, etc.) must be carefully addressed during the assessment of the structure. These determine the statical system upon which structural safety is assessed.

Further interesting and useful information may be obtained from a study of the structure's history: What happened during the lifetime of the structure may be important in assessing the safety of a structure, especially hazard scenarios that the structure may have successfully withstood in the past.

c) Loads and actions

On the basis of the updated service criteria agreement and the safety plan, expected loads and actions likely to occur during the residual service life must be specified. These might be quite different from what was assumed when the structure was planned. Some loads may have been considerably increased (see, e.g., with respect to snow loads, section 1.23).

For a short residual service life it may be appropriate to reduce environmental loads and actions (snow, wind, earthquakes etc.). For such interpolations one normally resorts to probability papers (see section 2.43), and in the case of climatic effects often to the Gumbel distribution.

Whereas snow and earthquake loading can be retained as such on the abscissa of the paper, it is reasonable to update wind speeds and to convert these to wind forces later.

Updating is also necessary for dead and permanent loads. The latter can often give rise to surprises (additional layers, partition walls not considered etc.). Naturally, corresponding safety reserves (load factors) cannot be left out, but, in view of eliminated uncertainties, may be slightly reduced.

Likewise, live loads on bridges, in warehouses, factories, etc., have to be updated. By paying proper attention to clear service instructions and adequate supervision, the exceedance of loads taken into account in the re-analysis can be prevented. The load factors, however, should in general not be reduced.

d) Material properties

At the time when the structure to be assessed was designed, some uncertainties about properties of the materials existed. The corresponding code, therefore, included some reserves on the safe side, mainly by specifying safety factors.

Now, at the time of the assessment of the structure, some of these uncertainties have naturally disappeared. Much better knowledge of the material properties can be gained by taking test samples from the structure. It is appropriate to include this improved knowledge in the assessment and moreover to reduce some of the safety reserves. On the other hand, the observations regarding corrosion, fatigue, wear, increased brittleness etc. should be included in the assessment.

It should be noted that the information gained from a few samples taken from the structure is generally not very great. The resulting values may be regarded only as some additional information, which has to be supplemented by all that is known from prior experience.

It is useful to plot the results from samples taken from the structure together with the prior information on suitable probability paper, in order to gain the necessary insight into the relevant material properties.

Often the source of structural or reinforcing steels, e.g., based on delivery notes, can be identified. If the brand of a steel is known, then often quite reliable values of material properties can be obtained, e.g., by enquiring at the respective steel production plant or at the materials testing laboratories used at that time.

An updating of the material properties can be omitted if they can be obtained reliably from the construction documents and the codes valid at the time of construction.

e) Structural system

Also the statical systems employed at the design stage due to the limited computational aids available, often were simpler and thus more approximate. Using advanced statical models, e.g., by including spatial structural behaviour, often allows for activating structural reserves. Quite often, structures include so-called non-structural elements, which can be utilised in the re-analysis. A typical example concerns walls, which often contribute decisively to the stability of existing structures though they were not taken into consideration at the design stage. Of course, if these are introduced into the analysis, their function must be guaranteed over the residual service life of the structure.

Another example concerns de facto continuous frame structures, which at the time of the design were assumed to be simple beams connected by hinges. A continuity effect can, at least partially, be assumed in order to mobilise reserves.

Activating structural reserves, however, is not always possible. For example: a plate that appears to carry load in an orthogonal manner may be reinforced in one direction only and carries load accordingly. Also, structural elements loaded from adjacent structural parts may be too weak to permit an alternative load path. In some circumstances specific after-inspection may be useful or necessary in order to clarify questions that arise during re-analysis.

Of course, changes of the statical system due to damage, weakening, or other defects must be considered in updating as well.

f) Methods of analysis and design

Computational methods have also evolved since the design of the structure under consideration. Up until the seventies, it was usual in analysis to apply elasticity theory and to base dimensioning on, e.g., the bending moment envelopes. Today, analysis concentrates on critical loading situations, and, as a rule, applies the lower bound theorem of plasticity theory. These two distinct differences in design attitudes often carry reserves into the re-analysis.

Also well into the seventies structural elements were dimensioned on the basis of so-called admissible stresses. Today, usually the sectional forces are compared with the respective sectional resistances. Here too it is often possible to mobilise reserves.

It should be observed that some design models permitted in earlier codes have proved to be unsafe and today the requirements are stricter. Examples are, e.g., stability problems in steel structures and, in reinforced concrete structures especially, shear and punching problems.

5.44 Reliability assessment

For an *existing structure,* safety or the lack of it is not intrinsic but is rather an expression of a particular opinion (e.g., of an expert) regarding the situation as encountered. The opinion is influenced by the observable but often poorly investigated physical properties of the structure. Thus, every statement about the safety of an existing structure is, in a sense, subjective and reflects the state of knowledge of the person making the statement. Indeed, expert opinions often differ considerably. However, as a rule, in the course of discussions the views held by the experts tend to converge and experts can, eventually, even reach full agreement. Experience shows that though views are subjective in a sense, there is rationalism in the final decision. Note that there are also formal probability based rules to deal with expert opinions (see *Bedford & Cooke, 2003)*.

Statements about the safety of an existing building are the result of a detailed analysis of the state and behaviour of its structure. Structural safety, however, is not a property of the structure. What, for example, at first sight appears to be unsafe, may upon closer examination be found to be safe. The opposite, however, may also occur. The updating of information about a structure will influence the initially somewhat subjective opinion concerning structural safety. Thus, the safety of an existing structure is a matter of decision rather than of science. Reliability theory is the tool and a rational basis for preparing such decisions.

Section 5.3 explained that fixing absolute target values for β is somehow linked to all assumptions made during the analysis. Thus, comparing derived β's with absolute target values β_0 is often a matter of conflict, especially with building authorities who are expected to accept the result of a reliability assessment. In order to convince such critics, it is advisable to base the assessment of the safety of a structure on the *axiom* that a correct application of the valid codes and standards results in a safe structure. Or, stated in accordance with *Ditlevsen & Madsen*, *1989*, a specific structure that can be proved to have a reliability index β bigger than the respective index β_0 can be considered safe enough.

The reliability index β_0 representing the safety level of the existing codes, can be derived by proper dimensioning of the specific structure according to the existing codes and standards and by then assessing this hypothetical structure and setting the resulting reliability index equal to β_0. In finding β_0, a number of assumptions must be made about the parameters of the variables entering the analysis. Since these are only marginally adjusted, most of the assumptions in the comparison cancel each other out.

Hence, the safety assessment consists of three steps:

- properly dimensioning the existing structure (as if planning a new structure), considering the consistent set of relevant codes and standards,
- calculating the reliability index β_0 related to the *dimensions thus obtained*, considering the parameters of the variables assumed to lie behind the models and variables of the codes and standards applied,
- calculating the reliability index β related to the *actual dimensions, properties* and *loads* of the structure under consideration, introducing up-to-date models and carefully updated parameters of the variables. Activating all possible reserves extensively explained in section 5.43, often in several steps is the way to hopefully declare the structure under consideration safe.

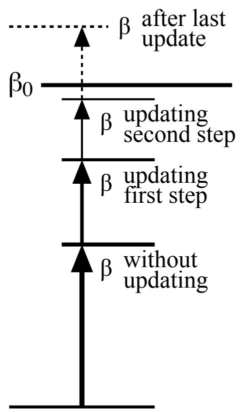


Fig. 5/15: Updating of information in search for safety

The structure may be considered safe if

$$\beta \geq \beta_0$$

If the comparison results in $\beta < \beta_0$, investing in further updating of dominant variables might be a good idea. In this respect also more advanced analyses (NL-FEM, FORM) may prove useful.

Finally one may decide to drop the value of β_0. This can be defended by a look at fig. 5/12 as in general the cost of reaching a higher level of reliability for existing structures may be quite prohibitive. Asking all existing building stock to meet actual code requirements would leave us bankrupt. Clearly, existing structures cannot always be evaluated by applying current code requirement. One may also try to apply non-structural remedial measures and reduce the consequences of failure. Note that in the end always some minimal levels of safety to life and limb of people should be obeyed. Applications of those principles may, e.g., be found in *SIA Technical Note 2018, 2004,* including operational thresholds for unity checks.

It is obvious that the absolute values of β and β_0 largely depend on the parameters introduced and as such are not comparable to other structures or other codes. The comparison between the two values β and β_0, however, reflects the influence of updated knowledge about the parameters under consideration. For an example see *Schneider, J., 1992.*

5.45 Strengthening structures, or weakening?

In case all tries to prove adequate reliability of an existing structure fail, it comes to think about strengthening. From an engineering point of view this, however, is a difficult task. In order to bring strengthening elements into force one first has to weaken the original structure. The co-operation between strengthening parts and the original structure is not always clear. Forces, after the change, may go unexplored paths and form a challenge for until then good functioning parts of the structure.

Sometimes, the opposite is also an option: the explicit weakening of a structure especially for strong dynamic types of loading (e.g., earthquake). Allowing for plastic deformation of some structural elements may hide essential parts of a structure from deadly overload.

A wealth of information and case studies is contained in *IABSE SED 12, 2010*.

And finally: yes, demolition and building a new structure is also an option. Often at least foundations can be used again.

5.5 Human Error – a case for Quality Assurance

As was shown already in section 1.21, human error is the main contributor to the number of people injured and killed and to the damage of property. It goes without saying that efforts should be spent to reduce these sad side effects of human activity. The key to this endeavour is Quality Assurance.

5.51 Perceptions of the concept

There are two fundamentally different perceptions of Quality Assurance: The one is as old as the engineering profession itself and springs from the fact that everybody in this profession tries to do his/her best to create the quality our clients are looking for. Quality Assurance in this sense stands for "the application of a comprehensive set of measures and activities aimed at assuring desired qualities of the product in design, execution, manufacturing, installation, maintenance, repair etc.". This is how the IABSE Rigi Workshop in 1983 defined the term (*IABSE, 1983)*.

The other definition of Quality Assurance was introduced mainly in conjunction with the construction of nuclear power plants and the associated public concern. There is a formal definition worked out by the American Society of Mechanical Engineers' Committee on Nuclear Quality Assurance. This committee defined Quality Assurance as "...all those planned and systematic actions necessary to provide adequate confidence that an item or facility will perform satisfactorily in service". Pay attention to the wording: *"... provide adequate confidence ..."*.

Providing confidence obviously implies providing proof, and that, in turn, implies a lot of paperwork. What comes out of this definition is a rather formal perception of the concept, one very much related to handbooks, forms, rubber stamps and signatures. Most engineers are rather apprehensive with regard to these matters. In addition, merely providing confidence in something is not enough. Quality itself is the issue and should be provided. Basically, it is a question of substance versus form. And, clearly, the issue is substance, not form.

Quite similar definitions lie behind *ISO 9000*, a series of codes that were developed for mass production of items and services and therefore may not be very relevant for assuring quality in the building industry (see, for instance, *Jensen, 1994,* and *Tang et al., 1997)*.

5.52 Quantification of quality

If quality is requested, quality should somehow be quantified. How to quantify quality can lead to rather lengthy and ultimately fruitless discussions. It seems that the best answer is to measure quality as the complement to non-quality, i.e., as the complement to deficiencies and damage. The less is spent on repairs, deficiencies, and damage, the better the quality.

The aim of Quality Assurance is – with affordable expenditures – to keep non-quality, i.e., deficiencies and damage, within acceptable limits. The questions are:

- How can this be done?
- Where is the best place to attack the problem? and
- Who, finally, is going to fight Human Error?

5.53 The battle against errors

The traditional weapon against errors is "checking", e.g., checking numbers, analytical operations, the conformity to codes, the strength of millions of concrete cylinders all over the world, the number of signatures on drawings – in short: checking anything that can easily be checked and that lends itself to quantification.

Clearly, the weapon "checking" has become blunted through unreflective and improper use. As a result, bureaucracy advances and the initiative and readiness of engineers to respond stagnates. The engineering profession is threatened by frustration and a decline of professional pride, confidence, and prestige.

A careful look at error-prone areas and phases of the building process and a thorough investigation of the characteristics of errors committed is certainly critical for a promising fight against errors. This research in large part is individual in the sense that everybody should carefully watch his/her own error characteristics and search for ways out. What applies to the individual also applies to teams working together or to different sectors of the building industry.

Four more general keywords may help to guide the way:

- *Motivation:* much more freedom for individual action should be given to the really dedicated people, within clearly defined areas of responsibility, of course.
- *Simplification:* error-prone concepts, systems, structural forms, and organisational schemes must be avoided.

- *Relaxation:* unnecessary constraints in matters of time, schedule, and money must be deleted.
- *Control and checking,* again: but these should be consciously applied at strategically well-chosen and effective places.

Some people believe that Quality Assurance is just a new burden on the shoulders of the construction industry, does not help much, and costs too much. This opinion may be true as long as Quality Assurance is performed in a bureaucratic way. It certainly is wrong if Quality Assurance measures are conducted in a way that is thoroughly attuned to the characteristics of the building process.

5.54 Do Quality Assurance efforts pay dividends?

From the review of the 800 failures reported in section 1.21, it became clear that 60% of the non-quality costs could have been avoided. These non-quality costs include the total expenses necessary to compensate damage to life, limb, and property and to compensate for deficiencies originating from faulty planning and execution, including openly negotiated and all so-called hidden costs.

From interviews it became clear that non-quality costs constitute between 5 and 10 percent of the project costs (see also *Gorisse & Declerck, 1985)*. These rather big numbers are not at all astonishing because all efforts made and time spent on correcting errors both in design offices and on the site are included here. The following considerations are based on an estimate of 5% for non-quality costs.

Investigations show that some 35% of the non-quality costs could be avoided without almost no additional activity other than the adequate attention of each subsequent partner in the building process. For example, the engineer should tell the architect immediately about a possible error in the documents received from the architect. As well, the contractor should advise the engineer in case he has questions about missing reinforcement bars.

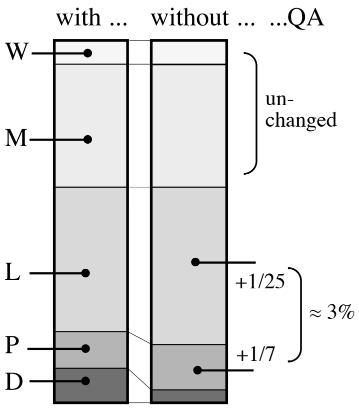


Fig. 5/16: Investing into quality assurance pays off

15% of the costs of non-quality are practically unavoidable. The remaining 50% may be detected through additional measures. On the assumption that there is success in half of the latter, a realistic estimate is that 60% of 5%, that is some 3% of the project costs, could be saved by a more conscious application of Quality Assurance measures and by adequate care being taken by those involved in the building process.

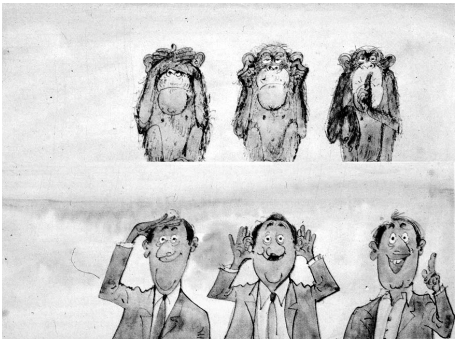
As to the costs: adding one additional person to a team of seven persons involved in planning, design, and management would certainly result in better quality. Adding that person would costs an additional 15% of the costs P related to the team. Because the costs of planning, design, and management P may be somewhere in the range of 10% of the project costs, this additional person would lead to an increase of the overall costs of about 15% of 10%, which is 1.5%.

The same consideration applies for the execution. Adding one person to a team of 25 people on the site, thus allowing the team to try to achieve better quality, would increase the manpower costs L involved with the execution by some 4%, which itself is estimated to be in the range of 40% of the execution costs. This extra person adds another 4% times 40%, i.e., some 1.5% to the project costs.

Thus, one additional person per seven in planning, design, and management of the project and one additional person per 25 on the building site would costs roughly 3% of the project costs. This is exactly the previously derived estimate of a possible economic benefit of more conscious application of Quality Assurance measures. In addition, the reduction in terms of human stress introduced by these extra personnel could well lead to an even greater benefit.

These examples provide ample evidence that applying Quality Assurance measures pays off. Starting to proceed along the lines proposed here would certainly reveal and correct additional weaknesses in the building process: weaknesses such as complicated organisation and information flow, unclearly defined competences, insignificant checks, unbalanced requirements, unclear aims and objectives. This all costs time, vexation, and in the end additional money. An elimination of all these deficiencies would definitely make the cost-benefit relation of Quality Assurance positive.

The *case* is that Quality Assurance in this sense is a matter of individuals mutually interacting and agreeing within a building process rather than a subject put forward by building authorities and enforced via codes and regulations. It is not a new and additional burden placed on the shoulders of the building profession but a new basic attitude, improving in a positive sense the necessary cooperation of all people involved.



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Don't shut your eyes, ears and mouth, as the *three wise monkeys* do.

Rather …

Stay alert!

This is the authors message to their readers!

6. Outlooks

In the previous chapters the presentation of the theory and in particular the mathematics have been kept as simple as possible. The advantage of such an approach is that it is very accessible, the disadvantage is that many useful applications of the theory cannot be explained. In this section, however, an outlook to more advanced applications will be presented, in the hope that readers will become enthusiastic and find their ways to more advanced literature.

6.1 Probabilistic analysis and Finite Element Models

Finite Element Models (FEM) is a quite common tool for engineers to assess the response of a structure under various static as well as dynamic loading conditions. As a FEM calculation is cumbersome in itself, the use in combination with reliability theory needs to be quite limited.

The outcome of a FEM calculation depends on the discretisation scheme. A finer mesh, as a rule, will lead to more accurate results. This type of (in)accuracy should also be taken care of, in addition to the randomness of loads and structural properties. It adds to the uncertainty of the results, but will not be elaborated further here.

6.11 Semi-probabilistic static analysis

a) Standard practice: linear analysis

In practice one usually undertakes a semi-probabilistic static linear analysis for every individual load case and next find stress resultants in every critical point by superposition using the combination rules specified in the code. If all *unity checks* (defined as the generalised stress resultant divided by the corresponding resistance value) are below 1.0, the structure is considered to meet the safety standards.

Even though this may look simple and straight forward, there are still debates on, e.g., the value of the Modulus of Elasticity: either mean value, characteristic value or design value and (in the case of concrete) cracked or uncracked condition. Other points of discussion are the proper inclusion of dynamic effects in case of wind or earthquake loading and the proper schematisation of structural connections and boundary conditions, both physically and from a reliability point of view.

b) Nonlinear analysis

If for a certain structure a linear analysis is considered as inadequate, nonlinear FEM analysis may be used to get a less conservative estimate of the load bearing capacity of the structure. Note that superposition in nonlinear analysis is not possible anymore, so one has to perform the calculation for all possible hazard scenarios. The recommended strategy is to do a linear analysis first and restrict the nonlinear analysis to the scenarios for which the linear analysis gives unfavourable results.

The debate on the use of values for material properties in the semi-probabilistic analysis is now extended to all yield and hardening or softening parameters of the material. Some experts prefer to use mean values for all random material properties and have a global resistance factor at the end; others prefer to use design values for all random properties. The problem is that it is not always known in advance whether a high or a low design value is on the safe side. The famous example is that low values for the bending capacity may lead to underestimation of the shear force.

If next to the physically nonlinear analysis also geometrically nonlinearities are included the standard codified buckling checks may be omitted. However, it may be difficult to include the proper imperfections (magnitude and direction) in the calculation.

6.12 Dynamic analysis

Another extension of the basic case is the dynamic analysis as is relevant for rapidly changing loads. This analysis may be done in time domain or frequency domain. In the time domain the load is specified as a time function and Newton's law is solved for each time step. In the frequency domain the loads are specified as a series of sine/cosine functions (Fourier series, Fourier transforms). The structure is analysed for each frequency and the results are added.

If the loads are random processes like for wind, waves, earthquakes, traffic, etc., commonly a frequency domain approach is followed using so called variance spectra (see 6.3). The spectrum of a structural response (for instance a stress) may be found by multiplying the variance spectrum of the load with the transfer function squared. The transfer function is defined as the ratio between a harmonic in- and output.

Given the spectrum for the stress at a certain point of the structure, the ultimate limit state and/or fatigue damage may be evaluated; serviceability may be judged from the output spectra for accelerations. In standard spectral analysis the structure is taken as linear and deterministic. However, both natural frequencies and damping in practice are highly uncertain. Again it is often difficult to judge in advance whether higher or lower values than the mean are unfavourable. If nonlinear analysis is of interest (like in seismic analysis) linearisation methods are possible but usually a time domain analysis is chosen. In that case a number of load signals are generated from the load spectrum (for instance 5 or 7). In some codes the mean result counts, in some the worst. The best evaluation, of course, is to use Bayesian estimates (see chapter 6.2) in combination with a proper probabilistic or semi-probabilistic analysis.

6.13 Stochastic FEM

At the highest level, a nonlinear dynamic time domain FEM analysis may be carried out, considering all loads, material properties, geometrical properties and model uncertainties as random variables, fields or processes. The probability of structural collapse or serviceability violation for the total design period of the structure should be considered. Note that some failure modes like, e.g., fatigue may require quite refined modelling at the scale of a connection or even weld.

In this way, of course, one avoids some of the problems encountered in semi-probabilistic analysis, like the choice of high, low or average design values for the various parameters. On the other hand, the combination of FEM and full probabilistic analysis can for the time being only be carried out if sufficient simplifications are introduced. Engineering judgement remains a key issues.

In stochastic FEM two types of discretisations are present: the classical one for the mechanical model and a second one for the stochastic model of the properties. There is no need for the two discretisation's to be the same. For the random fields one distinguishes between:

- Point discretisation (value at the centre of the element)
- Average discretisation (average value over the element)
- Series expansion methods.

The first two are self-explaining. Series expansion methods aim at expanding any realisation of the original random field over a finite set of deterministic functions with random coefficients. There are several series expansion methods, which are well explained in (*Sudret* and *Der Kiureghian, 2000)*.

Given the FEM model for the structure and the discretisation of the random properties, the actual analysis may start. For elastic analysis methods like Perturbation Techniques (*Nakagiri & Hisada, 1982)* and Neumann expansion solution (*Ghanem & Spanos, 1991)* may be used.

For the nonlinear elastic plastic analysis of frame structures where plasticity effects give relatively sharp changes in the stiffness behaviour, the so-called Branch and bound method is useful. It searches for an optimal solution by examining only a small part of the total number of relevant elastic plastic branches up to failure (*Murotsu, 1983*, *Thoft-Christensen* & *Murotsu*, *1986*).

By far the most popular method for dealing with complex nonlinear systems is the *Response Surface Technique* (*Bucher* & *Bourgund, 1990*). The idea is that the limit state function Z is generated by interpolation given a limited set of selected points in the *u*- or *x*-space (see fig. 6/1). The complex and time consuming FEM calculations are performed only for the selected points; the actual reliability analysis using FORM or Monte Carlo is based on the relative simple interpolation function. One option is to set up the response surface first and do the reliability calculations afterwards. Another option is to develop the surface as a part of the reliability procedure and to adapt the set of selected points in order to get better and more accurate results.

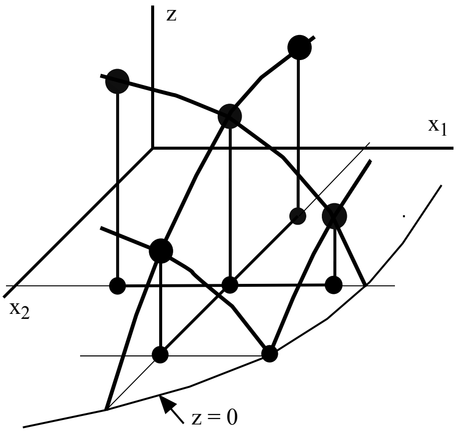


Fig. 6/1: Limit state function generated by interpolation between a limited set of selected points

An advanced and interesting calculation scheme by mixing Directional Sampling and an Adaptive Response Surface Technique (DARS) has been proposed by Waarts, (see *Waarts, 2000*). He proved that the total number of actually needed full limit state function evaluations may be comparable to those needed for standard partial factor verification.

6.14 FEM and model uncertainty

In classical structural analysis usually first the load effects in the structural elements (stresses, moments) are derived and then the safety verifications is performed on the member level. Model uncertainties in the structural analysis (see chapter 3 for an introduction) are then often related to these two steps.

In a FEM calculation it would make sense to relate the structural model uncertainties to the three basic groups of equations in the analysis:

1. The *geometric* equations, to find deformation (strain, curvature) given the displacements;
2. The *constitutive* equations, to find the stresses, moments, etc., given the deformations;
3. The *equilibrium* equations, to relate the internal forces with the external loads.

The steps 1 and 3 are related to each other through the mechanical principle of virtual work which means that also their model errors are connected. This means that in essence two types of model uncertainties should be considered:

(a) The model uncertainties following from the global schematisation of the structure (steps 1 and 3) like the neglect of 3D-effects, inhomogeneitys, interactions, boundary effects, simplification of connection behaviour, imperfections, etc.. The scatter of this model uncertainty will also depend on the type of structure (frame, plates, shell, continuums, etc.).

(b) The model uncertainties of step 2 that are related to the behaviour of a member, a cross section, or even the material in a single point. One may think in this respect of the visco-elastic model, the elastic plastic model, the yield condition (Von Mises, Tresca, Coulomb), the hardening and softening behaviour, the thermal properties and so on.

Current FEM programs, however, do not offer the option to enter model uncertainties in this manner. To incorporate the model uncertainties in the way described above, a change in the software offering an option to specify model uncertainty values may be required. The fall-back option is to incorporate (a) in the loads and (b) in the resistance parameters. For recommended values on model uncertainties, reference is made to the *JCSS Probabilistic Model Code*.

6.2 Bayesian parameter estimating

6.21 Basic Procedure

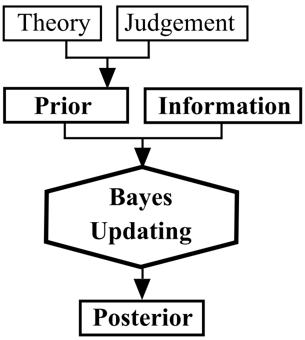


Fig. 2/3: Bayesian updating

The basic principle of a formal Bayesian updating has already been presented in chapter 2.14. Updating may be used for analysing the effect of new information on probabilities of related events. In this section the use of Bayes' Theorem will be discussed for estimating statistical parameters (e.g., mean, standard deviation) given a set of observations of the random quantity (strength, load, dimension). For the case of convenience Bayes' Theorem is repeated here, together with the respective figure:

$$P(B\,|\,I) = \frac{P(I\,|\,B)\cdot P(B)}{P(I)}$$

When applying the theorem to estimate statistical parameters, the event B represents some *hypothesis* on the parameter (e.g., the mean strength is 10 kN), while event I represents the available *information* or *data* (e.g., some observations). So Bayes' Theorem may be rewritten as:

$$P(\text{hypothesis} \mid \text{information}) = C \cdot P(\text{information} \mid \text{hypothesis}) \cdot P(\text{hypothesis}) \qquad (6.2)$$

In this equation P(I) = P(information) has been replaced by a constant C (or rather 1/C). This constant should be tuned in such a way that the resulting posterior probabilities add up to 1.0 as they should. It is therefore called a normalising constant.

Take, as an illustration, a random variable X, e.g., the resistance of a structural element. Based on long experience it is known that the distribution type is Gaussian and the standard deviation σ_X is 3 kN. The mean value μ_X is not known and will be derived on the basis of measurements. Assume that, before doing measurement, there is some prior knowledge on the mean that is expressed by a probability distribution $P(\mu_X = m)$. For instance one may feel that there is a 50% probability to have a mean value equal to 10 kN and another 50% probability that the mean value is 15 kN:

$P(\mu_X = 10) = 0.5$

$P(\mu_X = 15) = 0.5$

The units have been omitted for clarity of the formulas. In case the data is a set of observations having a sample mean m_X equal to some numerical value x, equation (6.2) can be further developed into:

$P(\mu_X = m \mid m_X = x) = C \cdot (m_X = x \mid \mu_X = m) \cdot P(\mu_X = m)$

where:

$P(\mu_X = m)$	=	prior distribution for the mean μ_X:
$P(\mu_X = m \mid m_X = x)$	=	posterior distribution for the mean μ_X
$P(m_X = x \mid \mu_X = m)$	=	likelihood of the data given the mean
C	=	normalising constant.

In words: the probability that the mean value μ_X equals m, given a sample average m_X, is equal to the product of the prior probability, the likelihood to obtain the sample mean x if $\mu_X = m$ and a normalising constant.

Let there be measurements on two arbitrary elements with an average value of 11 kN. Then using the above formulas:

$P(\mu_X = 10 \mid m_X = 11) = C \cdot P(m_X = 11 \mid \mu_X = 10) \cdot P(\mu_X = 10)$
$P(\mu_X = 15 \mid m_X = 11) = C \cdot P(m_X = 11 \mid \mu_X = 15) \cdot P(\mu_X = 15)$

As X is continuous, the probability of x being exactly 11 is simply zero and so one should actually write:

$P(\mu_X = 10 \mid m_X = 11) = C \cdot P(11 < m_X < 11 + dx \mid \mu_X = 10) \cdot P(\mu_X = 10)$
$P(\mu_X = 15 \mid m_X = 11) = C \cdot P(11 < m_X < 11 + dx \mid \mu_X = 15) \cdot P(\mu_X = 15)$

Let us elaborate the first line, the prior probability $P(\mu_X = 10) = 0.5$. To find the likelihood one should note that if $\mu_X = 10$ kN, the sum of two observations $(x_1 + x_2)$ has a mean equal to 20 kN and a standard deviation equal to $\sqrt{9+9} = 4.24$ kN; so the mean and standard deviation of the sample average $m_X = (x_1 + x_2)/2$ are 10 kN and 2.12 kN respectively. Given the normal distribution, the likelihood is given by

$$P = \frac{1}{\sigma\sqrt{2\pi}} \cdot \exp\left[-\frac{1}{2} \cdot \left(\frac{x-\mu}{\sigma}\right)^2\right]$$

resulting in

$$P(11 < m_X < 11 + dx \mid \mu_X = 10) = \frac{1}{2.12 \cdot \sqrt{2\pi}} \cdot \exp\left[-\frac{1}{2} \cdot \left(\frac{11-10}{2.12}\right)^2\right] \cdot dx$$

And so for the posterior probability that $\mu_X = 10$ one may find:

$$P(\mu_X = 10 \mid m_X = 11) = C \cdot \frac{1}{2.12 \cdot \sqrt{2\pi}} \cdot \exp\left[-\frac{1}{2} \cdot \left(\frac{11-10}{2.12}\right)^2\right] \cdot dx \cdot 0.5 = 0.17 \cdot C \cdot dx$$

In a similar way:

$$P(\mu_X = 15 \mid m_X = 11) = 0.03 \cdot C \cdot dx$$

C now may be found from the requirement that the sum of these two probabilities must be equal to one:

$C = 5/dx$

leading to:

$P(\mu_X = 10 \mid m_X = 11) = 0.85$
$P(\mu_X = 15 \mid m_X = 11) = 0.15$

To summarise: given that the average of two observations is 11, the probability of $\mu_X = 10$ has gone up from 0.50 (prior) to 0.85 (posterior). The probability that $\mu_X = 15$ has dropped from 0.50 (prior) to 0.15 (posterior).

6.22 General formula for updating / statistical uncertainty

If the parameter μ_X is not a discrete variable, but a continuous variable, or even a vector of parameters θ, equ. (6.2) may be written as:

$$f_\theta(\theta \mid I) = C \cdot f_X(x|\theta) \cdot f_\theta(\theta) \tag{6.3}$$

where:

$f_\theta(\theta|I)$ = posterior distribution for the parameters θ
$f_X(x|\theta)$ = probability density function for X, given θ (likelihood function)
$f_\theta(\theta)$ = prior distribution for the parameters θ
C = normalising constant

Graphically the Bayesian analysis for parameter estimation is presented in figure 6/2a for the general case and 6/2b for the non-informative case (see section 6.23).

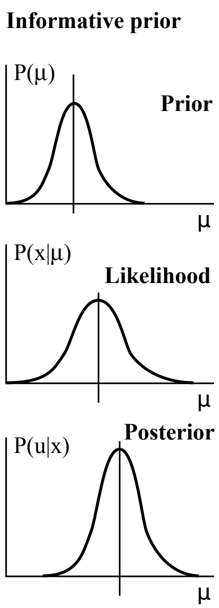


Figure 6/2a

The fact that one does not know a statistical parameter like the mean exactly, is referred to as *statistical uncertainty*. In the Bayesian approach there are two equivalent ways to deal with that uncertainty in the reliability analysis. One way is to consider θ as an additional random variable (or if more than one, a set of random variables). This is straightforward but increases the number of random variables.

The other way is to incorporate the statistical uncertainty into the distribution of the basic variables X, using the Theorem of the Total Probability (see section 2.13):

$$F_X(x \mid I) = \int F_X(x;\theta) \cdot f_\theta(\theta \mid I) \cdot d\theta \tag{6.4}$$

The distribution $F_X(x \mid I)$ is referred to as the *predictive* distribution. For further explanation, consider again the example of 6.21.

According to the first method described above, the Gaussian variable x in the limit state functions has explicitly to be replaced by the expression:

$x = \mu_X + u \cdot \sigma_X$

with u being a standard normal variable (zero mean and unit standard deviation), μ_X a random variable according to the (current) posterior distribution (in this example $P(\mu_X = 10) = 0.85$ and $P(\mu_X = 15) = 0.15$) and $\sigma_X = 3$ as before.

According to the second method described above the predictive distribution in the example arrives at:

$$F_X(x\,|\,I) = \sum P(X < x;\, \mu_X = m_k)\cdot P(\mu_X = m_k | x = 11) =$$

$$= \Phi[(x-10)\,/\,2.21]\cdot 0.85 + \Phi[(x-15)/2.21]\cdot 0.15$$

In this case there is, of course, a simple summation instead of an integration.

Note that the statistical parameters in the Bayesian approach are explicitly considered as random variables and treated in the same way as other random variables. In classical (or frequentistic) statistics, statistical parameters are treated as deterministic but unknown variables for which confidence intervals are specified. In Bayesian analysis, to put it simply, this confidence interval is already incorporated in the outcome

6.23 Non-informative or vague priors

As the choice of a prior may have great influence on the final result, very often also called '*vague*' or non-informative prior is used. This way the influence of the prior is tried to be kept at a minimum. Graphically this means that the priors in figure 6.2 are constant, flat and almost equal to zero. Two common examples will be discussed in the following.

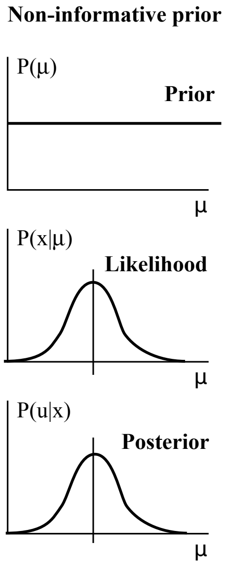


Figure 6/2b

(1) The variable X has a normal distribution with known standard deviation σ_X and completely unknown mean. In that case one may use the *non-informative* or *vague* prior as mentioned above. Skipping the mathematical details, the result is that the posterior distribution of μ_X has a normal distribution with mean equal to the sample mean m_X and a standard deviation equal to $\sigma_X\cdot\sqrt{(1/n)}$, where n is the sample size. From $x = \mu_X + u\cdot\sigma_X$ it can be inferred that the predictive distribution of x is normal with mean μ_X and standard deviation $\sigma_X\cdot\sqrt{(1+1/n)}$. So, for instance, in semi-probabilistic analysis, the 5% characteristic concrete strength $f_{5\%}$ in such a case would be:

$f_{5\%} = m_c - 1.64\cdot\sigma_c\cdot\sqrt{(1+1/n)}$,

wherein the term 1/n counting for the statistical uncertainty.

(2) Let x now be normal with mean and standard deviation both unknown. Assume that there is a set of n observations. It can be proven that on the basis of non-informative priors the posterior fractile for x can be found from:

$x_p = m \pm k_{n,p}\cdot s$

where:

m = sample mean
s = sample standard deviation
k = a coefficient to be taken from the following table (see *EN1990, Annex D)*

	n = 3	n = 6	n = 10	n = 30	n = ∞
p = 0.10	2.18	1.59	1.45	1.33	1.28
p = 0.05	3.37	2.18	1.92	1.73	1.65
p = 0.01	8.04	3.63	2.96	2.50	2.33

For instance: the 5% fractile of a concrete cube strength $f_{5\%}$ (the characteristic value), where a mean value of 30 MPa and a standard deviation of 4 MPa have been measured out of 6 cubes is equal to:

$f_{5\%} = m - 2.21 \cdot s = 30 - 2.21 \cdot 4 = 21.2$ MPa.

Note that if one has the same mean and standard deviation, but resulting from over 100 observations the result is $30 - 1.64 \cdot 4 = 23.4$ MPa. The difference between 23.4 and 21.2 is caused by the statistical uncertainty in the mean and the standard deviation.

6.3 Time and spatial variability

6.31 Fields and processes

If a number of samples from the soil on a building site or cores out of a concrete plate are tested, one will find that the strength varies form point to point. This would even be true if the measurement error would be eliminated completely. This spatial point to point variability makes that in principle one should consider the resistance as a random function of the spatial coordinates. Such a randomly varying spatial function is usually referred to as a stochastic field. To describe the field one needs to know the probability distribution for every point, but also the complete (auto) correlation structure.

A similar statement holds for variation in time. A random function of time is usually called a random or stochastic process. In fact this notion was already introduced in section 2.24 in relation to a river discharge. But the same holds for wind speeds, traffic flows, etc. Also here, for a complete description, the arbitrary point in time distributions for every point in time, but also the description of the (auto) correlation structure are needed.

Note that the correlation structure may be described directly by specifying the degree of correlation between any couple of points in space or time, but also indirectly by a functional description. The function $Y(t) = a + b \cdot t$,where a and b are independent random quantities, is a random process, simply because it is a function of time and a and b are random. One only needs the two dimensional statistical description of a and b to have a complete description of the process. Similar the function $Y(x) = a + b \cdot x$ is a (simple) random field.

In structural reliability in particular three types of random processes (or fields) are often applied:

- Rectangular wave processes,
- Poisson processes,
- Gaussian processes.

These processes are shortly discussed one by one.

6.32 Rectangular wave process

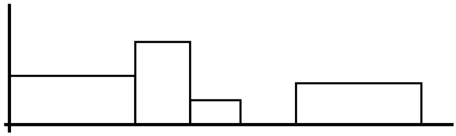


Fig. 6/3: Rectangular wave process

The rectangular wave process (or field) is relatively easy to deal with in a calculation. A number of varieties (see figure 6/3) may be distinguished:

- the time intervals may be random or deterministic,
- values in subsequent intervals may be dependent or independent,
- the process may be stationary or non-stationary.

The simplest example is the so called Ferry Borges-Castanheta model (FBC-model for short) that is characterised by:

- deterministic and equal time intervals Δt
- independent values of the process in the various intervals
- the same distribution functions $F_Y(y)$ for the value in every interval (stationary process).

The model for instance offers a very simple relation between the arbitrary point in time distribution and the extreme value distribution for some period T:

$$F_{Y\max}(y) = P\left(Y_{\max} < y\right) = P\left(Y_1 < y, \text{and } Y_2 < y, \text{and } ...\right) = P\left(Y_i < y\right)^n = F_{Y_{Y_i}}{}^n(y) \tag{6.6}$$

where:

Y_{max} = maximum or extreme value in period T
Y_i = arbitrary point in time value
n = $T/\Delta t$

The model is very convenient, for instance for load combination (see also section 3.43)

6.33 Poisson Process

The Poisson Process is in fact a special case of the rectangular wave process, with very short equidistant or even infinitesimal time intervals Δt and for each interval a large probability of being zero (see figure 6/4).

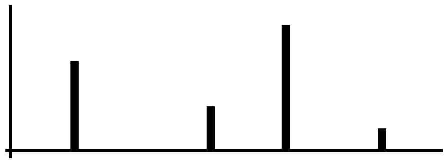


Fig. 6/4: Poisson process

The probability of having a non-zero value in an interval is $\lambda \cdot \Delta t$ with λ the Poisson occurrence rate. The non-zero process values may be random.

The model can be used for instance for modelling accidental loads like impact loads or explosions. Non-zero values are also used just to indicate that some kind of an event happens. The details of the event are then modelled outside the Poisson process. For instance one might say that the (average) occurrence rate λ of an earthquake event is once per 20 years or 0.05 per year. The details of the earthquake (peak acceleration, duration, mean frequency, etc.) can be described by a separate random model.

6.34 Gaussian process

Here the density function for all points in time is Gaussian. In order to describe such a process one needs the mean and standard deviation as a function of time and the correlation for each pair of points. If the mean and the standard deviation do not depend on time (and the correlation is only a function of the time gap) the process is stationary.

A very elegant model is to build up a stationary Gaussian process as the sum of a number of random sine functions. First consider the single sine function of time with amplitude a, angular frequency ω and random phase angle φ:

$$Y(t) = a \cdot \sin(\omega \cdot t + \varphi) \tag{6.7}$$

The stochastic part is exclusively confined to the random phase angle, which has a uniform distribution on the interval $(0, 2\pi)$. The value of the density function is $f(\varphi) = 1/(2\pi)$. The mean of Y(t) at an arbitrary point of time can be obtained from the standard definition:

$$\mu[Y(t)] = \int Y(t) \cdot f(\varphi) \cdot d\varphi = \int a \cdot \sin(\omega \cdot t + \varphi) \cdot (1/2\pi) \cdot d\varphi = 0 \tag{6.8}$$

In a similar way one can derive that the standard deviation is

$$\sigma[Y(t)] = a/\sqrt{2} \tag{6.9}$$

It may be concluded that this simple function is a stationary stochastic process, at least with respect to the mean and standard deviation. Next consider the summation of a number of sine functions with independent random phase angles:

$$Y(t) = \sum Y_k(t) = \sum a_k \cdot \sin(\omega_k \cdot t + \varphi_k) \tag{6.10}$$

The mean and standard deviation of the process simply follow from:

$$\mu[Y(t)] = \sum \mu[Y_k(t)] = 0 \tag{6.11}$$

$$\sigma[Y(t)]^2 = \sum \mu[Y_k(t)]^2 = \sum a_k^{\,2} \tag{6.12}$$

In applications, the number of sine functions in (6.10) depends on the accuracy required. Fig. 6/5 shows an example of such a process, which is built up out of 10 sine functions. The process may be called Gaussian according to the central limit theorem (see 2.62). The properties of this process are generally described using a continuous function of the frequency, called the spectral density function. If the spectral density is denoted by $S_{YY}(\omega)$, the values of a_k may be found by:

$$a_k^{\,2} = S_{YY}(\omega_k) \cdot \Delta\omega \quad \text{where } \Delta\omega = \omega_{k+1} - \omega_k \tag{6.13}$$

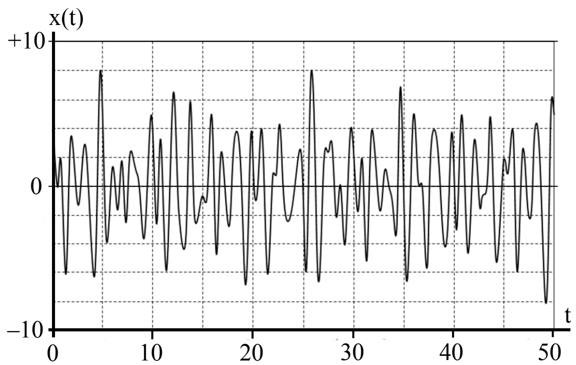


Fig. 6/5: Gaussian process

The Gaussian process shown in figure 6.5 is a simulation based, according to (6.4), on 10 unit amplitude sine functions with ω_k = 4.1; 4.3; 4.5; …; 5.9 rad/s and random phase angles φ_k.

The Gaussian process is used in practice for instance, to describe the wind velocity in one storm period (say 6 hours), or the waves in a single sea state (also about 6 hours), or the strong motion acceleration of an earthquake (say during 30 seconds). The advantage of this model is that if the response of a system to a single harmonic input is known, one may simply calculate the response to a random input. Note that also for Gaussian models relations between the arbitrary point in time a.p.t. (see section 2.24) and the extreme values can be derived. Formulas, however, are more complex than for the FBC-model.

For a complete description of the wind load one needs the superposition of an FBC-model (for the long term average wind speed) and a Gaussian model for the short term gusts. A similar statement holds for waves. For the description of earthquakes a Gaussian short term model may be combined with a Poisson model for the seismic events.

Gaussian fields are used for instance to describe the short distance random fluctuation of soils and concrete. Relevant is the ratio between the scale of the mechanism and the scale of random fluctuation. For instance, local soil variations are more important for a single pile foundation than for a large slip surface. A similar statement, by the way, holds for the loads: the wind velocity does not only fluctuate in time, but also in space. On a small area (e.g., a window) gusts may induce high peak pressures, but on a whole building these small scale fluctuations average out. This explains the area reduction coefficients for wind loads in codes of practice.

6.35 An application in foundation engineering

A contractor has to make a bid for producing a large number of piles on a construction field. The cost of a pile depends on the depth to the solid strata, and that depth is random. In order to make a rational bid the contractor wants to know the mean value and the scatter in the sum of all the distances to the solid.

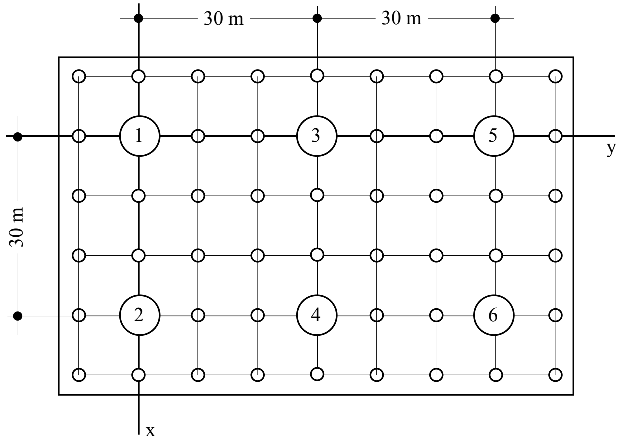


Fig. 6/6: Searching the depth of solid strata

Let the depth of the stratus (and thus the required pile length L_p) for a given coordinate system (x,y) be modeled as:

$$L_p = a + b \cdot x + c \cdot y + u \cdot s$$

where a, b, c, and s are constants and u is a zero mean, unit standard deviation Gaussian random field with a correlation function having one parameter d (the correlation distance or, as Erik Vanmarke put it, the scale of fluctuation). The correlation between u-values in two points (i) and (j) is then given by:

$$\rho(i,j) = \exp[-(R/d)^2]$$

where R is the distance between two piles at the locations i and j.

$$R = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}$$

Note that the value of d roughly corresponds to the reciprocal value of the averaged ω_k in (6.10).

Suppose first that the values of a, b, c, s, and d are known exactly. In that case one may quite easily calculate the probability distribution function of the sum $L = \Sigma L_p(i)$. However, in practice the values of a, b, c, s, and d may be not completely known.

In view of this, the contractor decided to use a cone penetration test at strategic points marked with numbers 1 to 6 in fig. 6/6. He arranged for a meeting between a geotechnician familiar with the area in order to collect prior knowledge and a person well known for his ability to make use of this towards a probabilistic basis for the bid. He felt pretty sure about how to arrive at a reliable bid on the basis of reliable information of the depth of the stratus at these six points.

In that case, following a Bayesian procedure, one should start by formulating a prior distribution for the 5 statistical parameters a, b, c, s and d. One might choose vague priors, but better is to use the knowledge of the geotechnical engineer. Given the test results, the priors can be updated to get the posterior distributions. The contractor may then make his bid on this updated information. The more piles he tests, the lower the resulting scatter and the more accurate the bid will be. However, also testing costs money and so one has to seek for an optimum.

The geotechnician was pretty sure about the depth of the strata at point 1. His suggestion was that it was between 8 and 12 m. He also pointed to his belief that the strata was going somehow deeper towards point 6 and that at that point the strata might well be as low as 20 m. His suggestion was to probe at points 1 and 6 first and then possibly continue at point 2 and 5.

The probability expert translated this information into the following prior information:

$a = N(10\text{ m}; 0.7\text{ m})$

$b = c = (20 - 10)/(60+30) = 0.11$, $d = 30$ m and $s = 1.0$ m (considered as deterministically known for convenience)

To simplify the problem further for the case of this example, assume that the length of pile 1 is equal to the length of all 3x3 piles where pile 1 is the center. Similar assumptions can be made for piles 2 to 6. Readers may imagine that the solution of a problem without this simplification requires more calculation efforts, but runs in principle according the same lines.

If *no test* at all is performed the situation is the following (again, the units have been omitted for clarity of the formulas):

- The mean value of the sum of the length of all 6 pile groups of 9 piles is:
  $\mu(L) = 9 \cdot (10 + 13.33 + 13.33 + 16.67 + 16.67 + 20) = 810$ m
- The standard deviation from the u-field follows from a summation over 6 x 6 terms:
  $\sigma(L)^2 = 9^2 \cdot \sum\sum s^2 \cdot r(i,j) = 81 \cdot 1.0^2 \cdot (3.51)^2 = (31.6)^2$
  The factor 9^2 follows from the presence of 9 piles in one group
- And, finally, all 9 x 6 piles have a correlated scatter following from the uncertainty in the parameter a, which is simply $\sigma(L) = 9 \cdot 6 \cdot 0.7 = 37.9$;
- In total the uncertainty adds up to $\sigma^2(L) = (31.6)^2 + (37.9)^2 = (49.3)^2$.

This means that the total length could vary roughly between 700 and 900 m.

Now suppose the contractor *tests pile (1)*, having coordinates x = y = 0. Let the test result be $L_{p1} = 11$m. In that case one can update the distribution for the parameter a using the Bayesian procedure:

$f''(a) = C \cdot P(L_{p1} = 11\text{m} \mid a) \cdot f'(a)$

where f' stands for the prior and f'' for the posterior.

f '(a) is the prior normal distribution for a with mean 10 m and standard deviation 0.7 m and the likelihood of the data is given by:

$$P(L_{p1}=11|a)=\frac{1}{\sqrt{2\pi}}\cdot\frac{1}{s}\cdot\exp\left[-(L_{p1}-a-n\cdot x-c\cdot y)^2/2s^2\right]=\frac{1}{\sqrt{2\pi}}\cdot\frac{1}{s}\cdot\exp\left[-(11-a)^2/2\right]$$

The posterior f ”(a) is obtained by multiplication of the prior and the likelihood and then using C to normalize ∫ f”(a)da on 1.0. The result is that the posterior is a normal distribution with mean 10.3 m and standard deviation 0.57 m. The updated situation for the mean of L is now:

$\mu(L) = 9 \cdot (11 +13.63+13.63+16.97+16.97+20.3) = 833$ m

The length of the first pile group is directly based on the measured value, the others have been calculated on the basis of the updated mean value of a.

In the calculation of the standard deviation one has only to look at the pile groups 2 to 5 as the length of pile group 1 is now deterministic, so:

$$\sigma(L)^2 = 9^2\cdot\sum\sum s^2\cdot\rho(i,j)+(9\cdot5\cdot0.57)^2=(27.8)^2+(25.8)^2=37.9^2\ \text{m}^2$$

So the mean value of L raises from 810 to 833 m and the scatter reduces from 49.5 m to 37.9 m.

If the contractor would decide to test a second pile (*say pile 6*) and find a result $L_{p6} = 21$ m, in a similar way

$\mu(L) = 845$ and $\sigma(L)^2 = (25.1)^2 + (17.9)^2 = 30.8^2$

can be found.

Note that when elaborating the likelihood care should be taken of the correlation between piles 1 and 2. Whether this has helped the contractor to make a better bid and whether that was worth the effort is outside the scope of this example. Note that in this case all uncertainty can be removed by having tests on 6 locations.

6.36 An application in wind engineering

Wind is a stochastic process. During periods of a few hours it can be considered as stationary and Gaussian process. So wind may fully be described for such a period (say one hour for convenience) using a mean value μ, a standard deviation σ and a spectrum S. The coefficient of variation σ/μ is in wind engineering referred to as the turbulence intensity I. For design it is important to know the expected maximum wind velocity in that hour.

To some extent a Gaussian process may be conceived as a sequence of random peaks. From more advanced theory it is known that the individual peaks of a narrow band process follow a Rayleigh distribution:

$P(v_{peak} > x) = \exp(-\beta^2/2)$ where $\beta = (x - \mu)/\sigma$

Now make the incorrect but useful assumptions that

- the wind process is narrow banded and
- all peaks are independent.

In that case one may write for the maximum velocity v_{max} in one hour:

$P(v_{max} > x) = 1 - [1 - P(v_{peak} > x)]^n \approx n \cdot P(v_{peak} > x) = n \cdot \exp(-\beta^2/2)$

where n is the number of peaks in one hour, roughly equal to

$n = T \cdot f_0 = T \cdot (2\pi\cdot\omega_0)$

wherein T = 1 h = 3600 s and ω_0 the central frequency of the spectrum defined from:

$$\omega_0{}^2 = \frac{\int \omega^2 \cdot S \cdot d\omega}{\int S \cdot d\omega}$$

However, for theoretical turbulence spectra this value tends to infinity. In structural engineering actually only frequencies up to the main natural frequencies are of importance. Taking the first natural frequency in most cases is a good approximation as a central frequency.

We are looking for the expected maximum of the distribution of v_{max}. It is however easier to take the median value instead of the mean (the error is small). In that case v_{max} follows from:

$$v_{max} = \mu + \beta \cdot \sigma$$

and β follows from

$$n \cdot \exp(-\beta^2/2) = 0.5.$$

In wind engineering the β is referred to as the gust factor and therefore usually the symbol k instead of β is used:

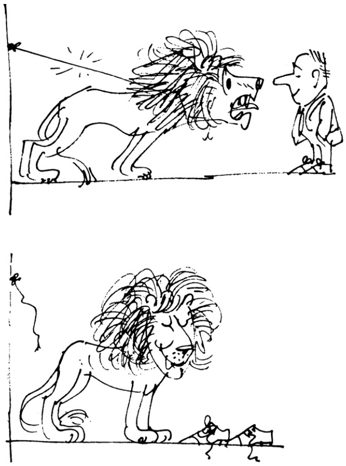
$$k = \sqrt{2 \cdot \ln(2 \cdot T \cdot f_e)}$$

Just as an numerical example: Take a one hour period, so T = 3600 s and take as natural frequency f_e = 1 Hz. This leads to a gust factor k = 4. Eurocode EN1991-1-4, annex B.2, for comparison, uses k = 3.5.

This means that structures are designed for a wind speed $v_{max} = \mu + 4 \cdot \sigma = \mu \cdot (1+ 4 \cdot I)$, where μ is the (hourly) mean wind speed with a return period corresponding to the required reliability and I is the turbulence intensity. The term $4 \cdot I$ takes into account the wind speed fluctuations within the period of one hour. Strictly speaking it is a bit too low as it is based on the expectation and so higher values are likely; further storms may last longer than one hour and there is more than one storm in the life of a structure.

7. Appendix

Just as a joke (or is it a warning?) here something to think about:



Cartoon by courtesy of *Dicke, 1975*

7.1 Murphy's Law

"*If anything can go wrong, it will!*"

O'Toole (whoever this was) replied by:

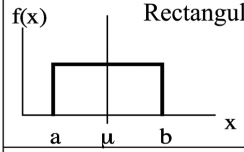
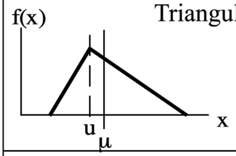
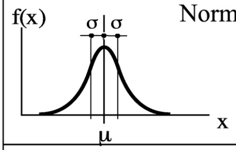
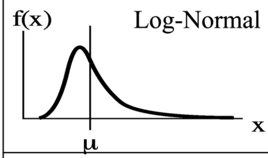
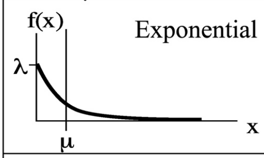
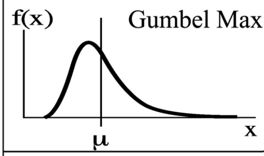
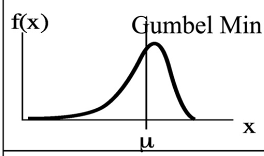
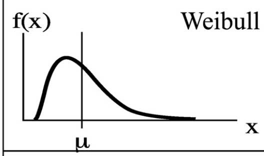
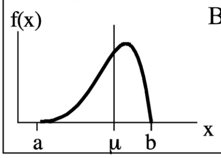
"*Murphy was an optimist.*"

Corollaries:

- Nothing is as easy as it looks.
- Everything takes longer than you think.
- If there is a possibility of several things going wrong, the one that will cause the most damage will be the first one to go wrong.
- If you perceive that there are four possible ways in which a procedure can go wrong, and circumvent these, then a fifth way will promptly develop.
- Left to themselves, things tend to go from bad to worse.
- Whenever you set out to do something, something else must be done first.
- Every solution breeds new problems.
- It is impossible to make anything fool proof, because fools are so ingenious.
- Nature always sides with the hidden flaw.

See: https://en.wikipedia.org/wiki/Murphy's_law
and many other sources.

7.2 Frequently used distribution types

Rectangular 	$-\infty < a < b < +\infty$ $\mu = \frac{a+b}{2}$ $\sigma = \frac{b-a}{\sqrt{12}}$	$a \le x \le b$ $f(x) = \frac{1}{b-a}$ $F(x) = \frac{x-a}{b-a}$	
Triangular 	$-\infty < a < b < +\infty$ $\mu = \frac{1}{3}(a+b+u)$ $\sigma = \sqrt{\frac{1}{18}\left(a^2+b^2+u^2-ab-au-bu\right)}$	$a \le x \le u$ $f(x) = \frac{2}{b-a}\left(\frac{x-a}{u-a}\right)$ $F(x) = \frac{x^2-2ax+a^2}{(b-a)(u-a)}$	$u \le x \le b$ $f(x) = \frac{2}{b-a}\left(\frac{b-x}{b-u}\right)$ $F(x) = 1-\frac{x^2-2bx+b^2}{(b-a)(b-u)}$
Normal 	$-\infty < \mu < +\infty$ $\sigma > 0$ μ σ	$f(x) = \frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right)$ $F(x) = \frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{x}\exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right)dx$	
Log-Normal 	λ, ζ $\mu = \exp\left(\lambda + \frac{\zeta^2}{2}\right)$ $\sigma = \mu\sqrt{\exp\left(\zeta^2\right)-1}$	$f(x) = \frac{1}{\zeta x\sqrt{2\pi}}\exp\left(-\frac{1}{2}\left(\frac{\ln x-\lambda}{\zeta}\right)^2\right)$ $F(x) = \int_0^x \frac{1}{\zeta x\sqrt{2\pi}}\exp\left(-\frac{1}{2}\left(\frac{\ln x-\lambda}{\zeta}\right)^2\right)dx$	
Exponential 	$\lambda > 0$ $\mu = \frac{1}{\lambda}$ $\sigma = \frac{1}{\lambda}$	$x \ge 0$ $f(x) = \lambda\exp(-\lambda x)$ $F(x) = 1-\exp(-\lambda x)$	
Gumbel Max 	u, α $\gamma \cong 0.577216$ $\mu = u + \frac{\gamma}{\alpha}$ $\sigma = \frac{\pi}{\alpha\sqrt{6}}$	$f(x) = \alpha\cdot\exp(-\alpha(x-u)-\exp(-\alpha(x-u)))$ $F(x) = \exp(-\exp(-\alpha(x-u)))$	
Gumbel Min 	$\mu = u - \frac{\gamma}{\alpha}$ $\sigma = \frac{\pi}{\alpha\sqrt{6}}$	$f(x) = \alpha\cdot\exp(\alpha(x-u)-\exp(\alpha(x-u)))$ $F(x) = 1-\exp(-\exp(\alpha(x-u)))$	
Weibull 	$\varepsilon \le x < +\infty$ $k > 0$ $\mu = \varepsilon + (u-\varepsilon)\Gamma\left(1+\frac{1}{k}\right)$ $\sigma^2 = (u-\varepsilon)^2\left[\Gamma\left(1+\frac{2}{k}\right)-\Gamma^2\left(1+\frac{1}{k}\right)\right]$	$f(x) = \frac{k}{u-\varepsilon}\left(\frac{x-\varepsilon}{u-\varepsilon}\right)^{k-1}\cdot\exp\left(-\left(\frac{x-\varepsilon}{u-\varepsilon}\right)^k\right)$ $F(x) = 1-\exp\left(-\left(\frac{x-\varepsilon}{u-\varepsilon}\right)^k\right)$	
Beta 	$-\infty < a < b < +\infty$ $r,s \ge 1$ $\mu = a + (b-a)\cdot\frac{r}{r+1}$ $\sigma = \frac{b-a}{r+s}\cdot\sqrt{\frac{r\cdot s}{r+s+1}}$	$f(x) = \frac{\Gamma(r+s)}{\Gamma(r)\cdot\Gamma(s)}\cdot\frac{(x-a)^{r-1}(b-x)^{s-1}}{(b-a)^{r+s-1}}$ $F(x) = \frac{\Gamma(r+s)}{\Gamma(r)\cdot\Gamma(s)}\cdot\int_a^u\frac{(u-a)^{r-1}(b-u)^{s-1}}{(b-a)^{r+s-1}}du$	

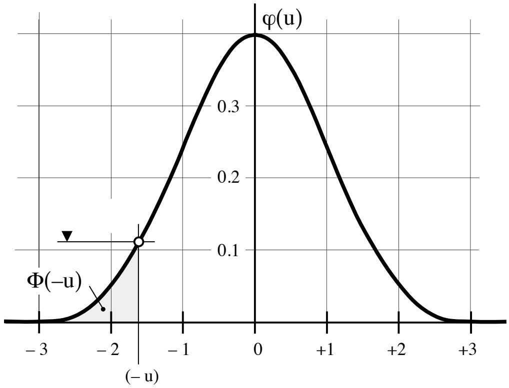
7.3 Standard Normal distribution

Definition

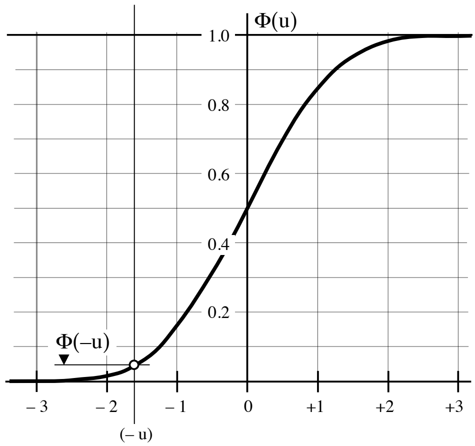
$$\varphi(u) = \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{1}{2}u^2}$$

$$\Phi(u) = \int_{-\infty}^{u} \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{1}{2}u^2} du$$

Probability density function, pdf



Cumulative distribution function, cdf



u	Φ(– u)	φ(u)
.0	.500 00	.398 94
.1	.460 17	.396 95
.2	.420 74	.391 04
.3	.382 09	.381 39
.4	.344 58	.368 27
.5	.308 54	.352 07
.6	.274 25	.333 22
.7	.241 96	.312 25
.8	.211 86	.289 69
.9	.184 06	.266 09
1.0	.158 66	.241 97
1.1	.135 67	.217 85
1.2	.115 07	.194 19
1.3	.096 80	.171 37
1.4	.080 76	.149 73
1.5	.066 81	.129 52
1.6	.054 80	.110 92
1.7	.044 57	.094 05
1.8	.035 93	.078 95
1.9	.028 72	.065 62
2.0	.022 75	.053 99
2.1	.017 86	.043 98
2.2	.013 90	.035 48
2.3	.010 72	.028 33
2.4	.008 20	.022 40
2.5	.006 21	.017 53
2.6	.004 661	.013 58
2.7	.003 467	.010 42
2.8	.002 555	.007 92
2.9	.001 866	.005 95
3.0	.001 350	.004 43
3.1	.000 968	.003 27
3.2	.000 687	.002 38
3.3	.000 483	.001 72
3.4	.000 337	.001 23
3.5	.000 233	.008 73
3.6	.000 159 1	.000 612
3.7	.000 107 8	.000 425
3.8	.000 072 3	.000 292
3.9	.000 048 1	.000 199
4.0	.000 031 7	.000 134
4.1	.000 020 7	.000 089 3
4.2	.000 013 3	.000 058 9
4.3	.000 008 5	.000 038 5
4.4	.000 005 4	.000 024 9
4.5	.000 003 4	.000 016 0
4.6	.000 002 1	.000 010 1
4.7	.000 001 3	.000 006 4
4.8	.000 000 8	.000 003 9
4.9	.000 000 5	.000 002 4

7.4 Probability plotting papers

You will find here the most important probability plotting papers. With respect to constructing probability papers reference is made to section 2.43. The use of these is described here shortly.

Probability plotting papers may be downloaded from http://www.weibull.com/GPaper/

7.41 Procedures

a) Choosing the most appropriate plotting paper

Normal distribution paper is chosen to hold data which are spread rather symmetrical to the left and the right of the mean. The paper allows for positive and negative values and is suitable for many kind of data, e.g., dimensions of structural parts, average point in time loads, water levels, etc.

Log-normal distribution paper is often chosen for data which cannot be negative. This holds for, e.g., strength data, concrete cover, etc.

Gumbel distribution paper is mainly chosen for the analysis and extrapolation of data like snow and wind load, etc.

b) Introducing the data

Each value of a sample on a probability paper is represented by a point. The co-ordinates of the points are obtained as follows:

- The values of the sample are put in a table in increasing order.
- Each value is assigned, corresponding to its relative position n within the sample, a value

$$F_X(x_n) = \frac{n}{N+1} \tag{7.1}$$

  and is written down in a further column. N corresponds to the number of measured values.

  If extreme values are of interest the following formula is recommended:

$$F_X(x_n) = \frac{n-0.4}{N+0.2} \tag{7.1a}$$

- Each value x_n of the sample is plotted as a point $(x_n; F_X(x_n))$ on the probability paper.

c) Fitting the straight line

The series of points thus obtained is approximated visually by using the most appropriate straight line. If the extreme values of the distribution are of interest, then more weight should be given to them. If *strong* deviations are observed, perhaps changing the type of probability paper will help. The procedure is repeated until a satisfactory approximation is obtained.

There are computer programs that do this job, giving equal weight to each of the data points. It must be stated, however, that some problems require more emphasis on the smaller, some on the larger values.

The chosen straight line, together with the type of probability paper, characterises the appropriate distribution. It is then only a question of deriving the respective distribution parameters.

d) Calculation of the distribution parameters

The calculation of the distribution parameters is based on the straight line found and makes use of two pivot points as follows:

- *Normal distribution:* one reads the values x_0 and x_1, which fix the chosen straight line for $F_X(x_0) = 0.5$ and $F_X(x_1) = 0.841$ on normal probability paper. From these values the mean value μ and the standard deviation σ are given:

  $\mu = x_0$

  $$\sigma = (x_1 - x_0) \tag{7.2}$$

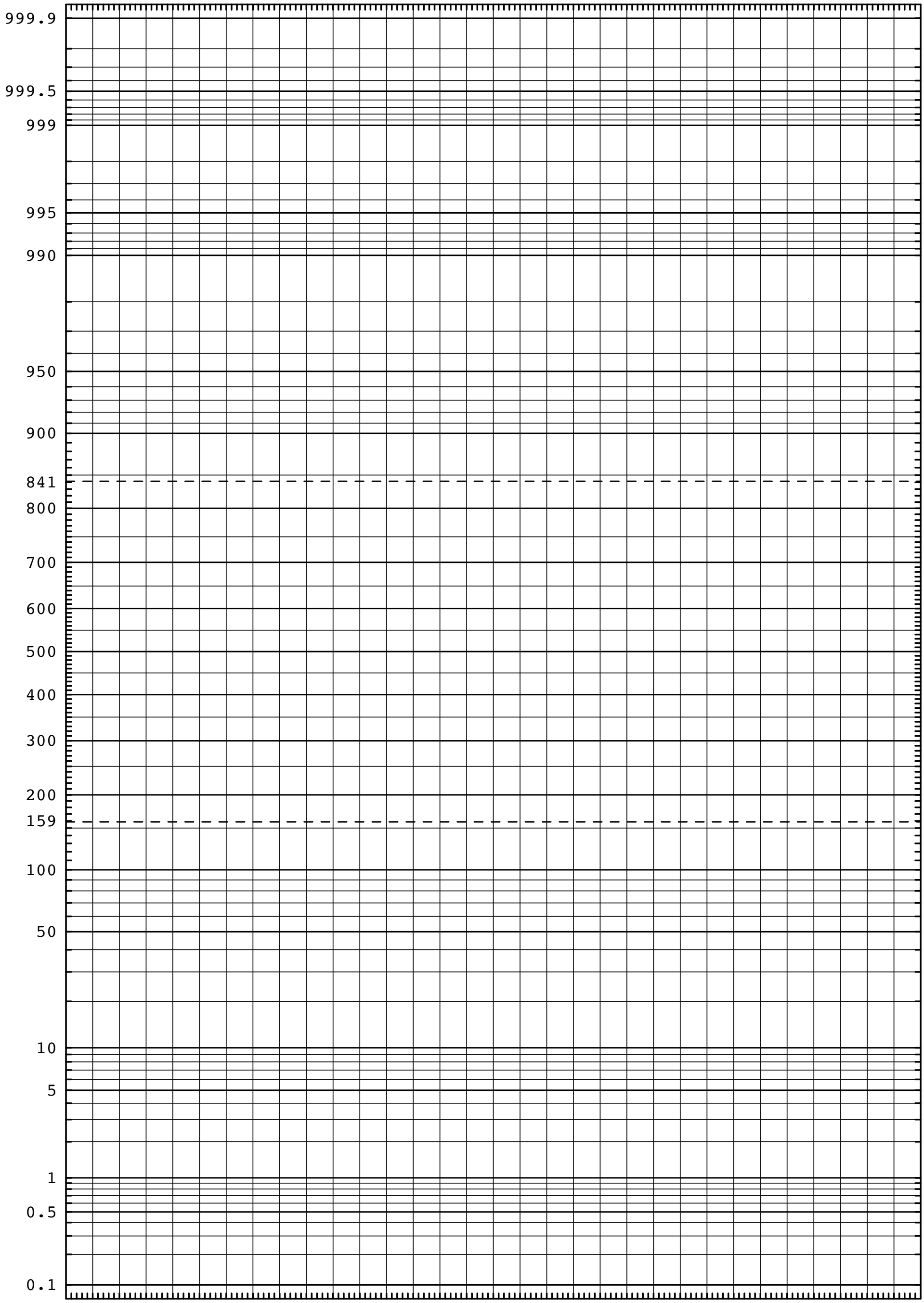
- *Log-normal distribution:* one reads the values x_0 and x_1, which fix the chosen straight line for $F_X(x_0) = 0.5$ and $F_X(x_1) = 0.841$ on log-normal probability paper. From these values the parameters λ and ζ are given, from which the mean μ and the standard deviation σ may be calculated:

  $$\lambda = \ln(x_0) \qquad \mu = e^{(\lambda + \zeta^2/2)}$$
  $$\zeta = \ln(x_1/x_0) \qquad \sigma = \mu \cdot (e^{\zeta^2} - 1)^{1/2} \tag{7.3}$$

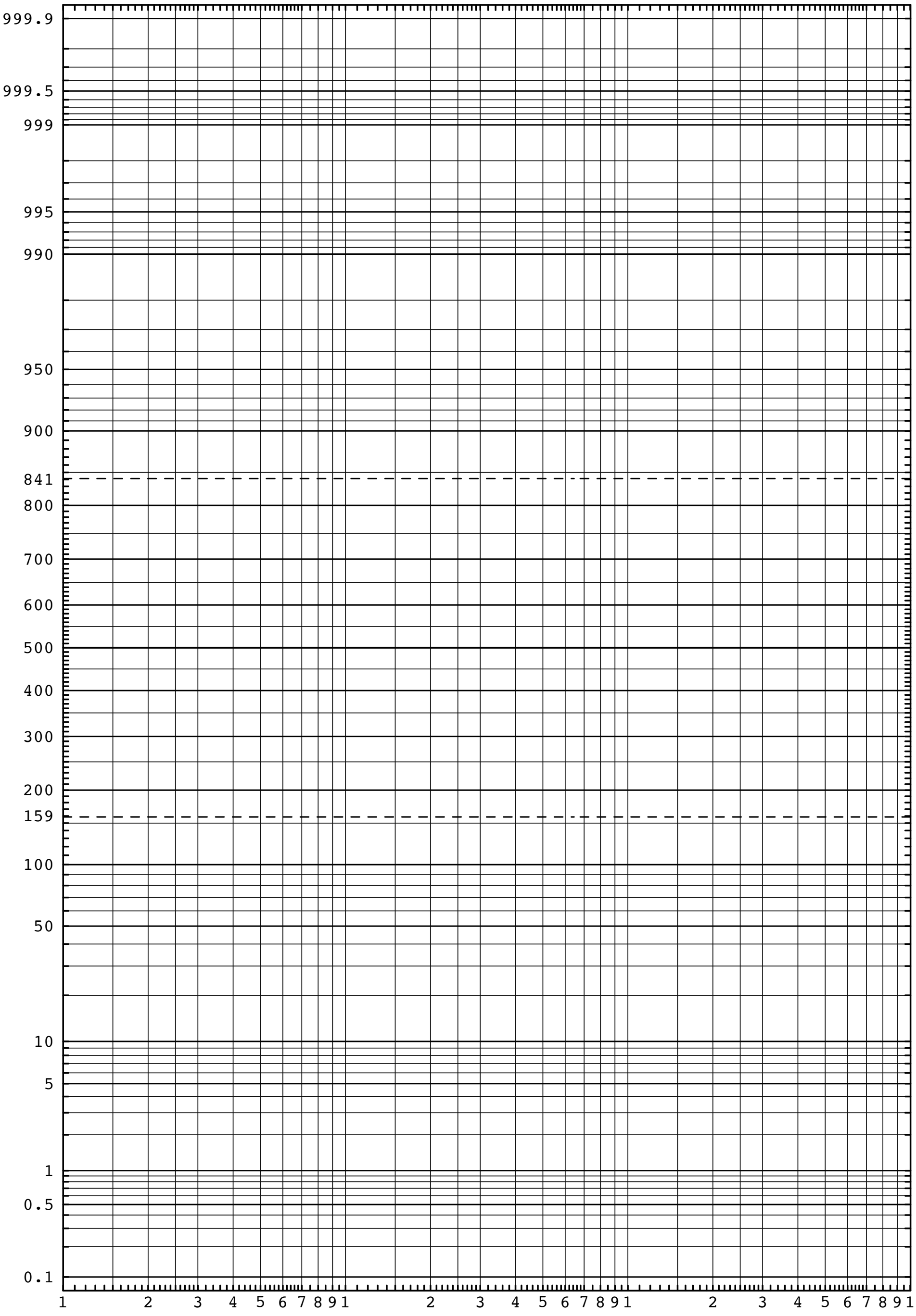
- *Gumbel distribution (T1L):* one reads the values x_0 and x_3, which fix the chosen straight line for $F_X(x_0) = 0.368$ and $F_X(x_1) = 0.951$ on Gumbel probability paper. From these values the parameters u and α are given, from which the mean value μ and the standard deviation σ may be calculated:

  $$u = x_0 \qquad \mu = u + 0.577/\alpha$$
  $$\alpha = 3/(x_3 - x_0) \qquad \sigma = 1.283/\alpha \tag{7.4}$$

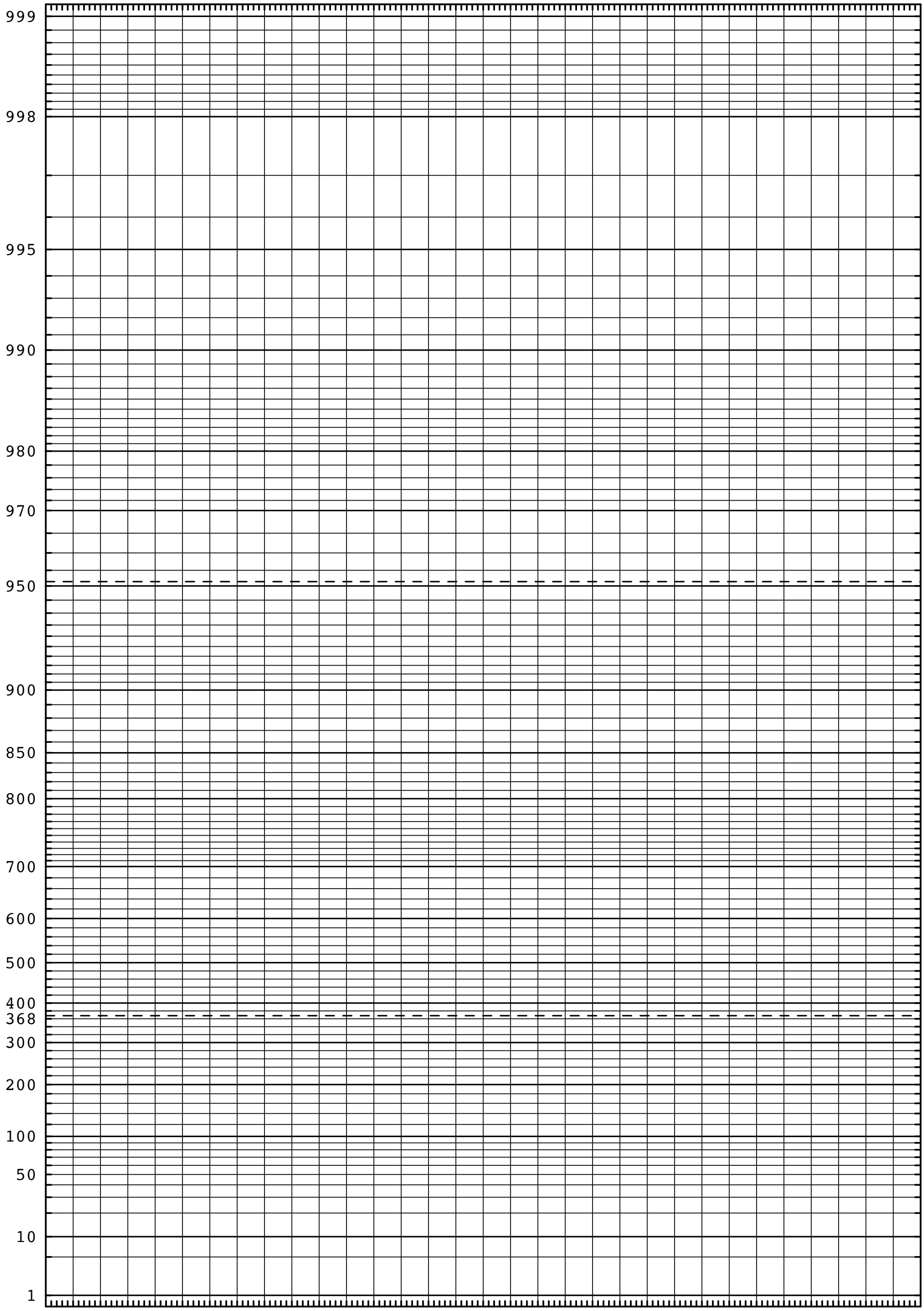
7.42 Normal probability plotting paper



7.43 Log-normal probability plotting paper



7.44 Gumbel probability plotting paper



7.5 Computer programs

There is a lot of computer programs and program packages that may be of use in analysing safety and reliability of structures and other technical systems. A very good overview on what is on the market is given in *Karadeniz & Vrouwenvelder, 2006*. Some of the more mature programs are described in the following.

7.51 STRUREL

In 2016, Dr. Stephan Gollwitzer, Managing Director of RCP Consult GmbH, wrote the following:

STRUREL is one of the most complete collections of software modules for probabilistic modeling in structural engineering. It offers state-of-the-art structural reliability computational methods. The definition of the probabilistic model is fast and efficient with Strurel: The user-friendly graphical interface (GUI) and the ad-hoc interpreter guide the user to run the probabilistic analysis and to process the results. Failure criteria (state functions) can either be implemented in the GUI or through the interface with external programs (MATLAB or user-defined DLL).

STRUREL includes the main package COMREL (-TI) and four extensions COMREL-TV, SYSREL, COSTREL and STATREL.COMREL performs time-invariant reliability analysis of individual failure modes based on advanced FORM/SORM methodology. Several algorithms to find the most likely failure point (β-point) are implemented including a gradient free algorithm for non-differentiable failure criteria (state functions). Complementary or alternative computational options are Mean Value First Order (MVFO), Monte Carlo simulation, Adaptive Sampling, Spherical Sampling, several Importance Sampling schemes and Subset Simulation.

COMREL-TV is the extension of COMREL to Time-Variant reliability analysis. The failure probability is computed by the outcrossing approach also based on FORM/SORM methodology for stationary or non-stationary cases. Available random process models are regular or intermittent rectangular wave processes and differentiable Gaussian and non-Gaussian translation processes (Hermite or Nataf processes). Both models can be scalar processes and vector processes.

SYSREL is the extension to cover system reliability evaluation. The logical model in SYSREL is connected with the failure criteria and the stochastic model for a fully interactive control. System modeling includes not only the representation by a (minimal) set of parallel systems in series but also the important case of conditional events (observations, event updating). For the FORM/-SORM methods SYSREL is based on, one has access to several efficient and reliable algorithms searching for the β-point with special solution strategies. An alternative computational option is Monte Carlo simulation.

COSTREL is the extension of COMREL for reliability-based optimization. Both optimization of an arbitrary objective (or cost) function under a reliability constraint as well as optimization (minimization) of failure probability for maximally admitted costs can be performed. COSTREL comprises two efficient optimizers, user definable starting solution and various algorithmic controls. The reliability part is based on FORM/SORM allowing to compute a rich set of sensitivity measures for both stochastic variables and cost variables.

STATREL covers standard statistical analyses and has many additional features for statistical reliability-oriented data analysis. For all models included in other STRUREL modules STATREL performs parameter estimation by different methods, confidence interval and quantile estimation as well as hypothesis testing including tests for sample validity, distribution functions and parameters. Simple analysis of variance and regression is also included. Several Bayesian methods are implemented. Results are made visible in terms of numerous graphical representations.

Developer and Distributor: RCP Consult GmbH, Munich, Germany

For more details see http://www.strurel.com

7.52 VaP and FreeVaP

In 2016, Dr. Markus Petschacher, wrote the following:

The program VaP, the name drawn from ***Va**riables **P**rocessor*, was developed at ETH Zurich by Markus Petschacher to support "stochastic thinking" (see *Petschacher, 1993)*, and was and is used mainly for educational purposes. Most of parameters and physical quantities are stochastic quantities. Their mathematical treatment has to be simplified.

Petschacher later developed his ideas further and wrote a new and much more complete version of VaP from scratch. The version available at the time of print, VaP 4.0 is a professional program lending itself to modern analysis in the fields of probability and reliability theory.

VaP 4.0 operates as interpreter and compiler for a wide range of problem statements. It parses stochastic variable notations, limit state functions, fault or decision tree formulations checks their syntactical and semantical correctness and produces on the fly executable internal code. That way high performance for simulation methods is archived. The range of implemented distribution types has grown consequently.

New areas are stochastic network and renewal reward theory analysis capabilities. The motivation behind is support for dealing with maintenance problems of bridges, road pavement and similar fields.

For analysis, the FORM/SORM approach as well as different simulation methods can be used. The output is stored in a browser like environment and can be easily copied into a user prepared report. For NP hard problems like network analysis advanced combinatorial schemas are implemented.

VaP 4.0 is a DotNet application written completely in C#. External software components can be loaded during runtime and used to extend the capacity of the software. The capabilities of this software are best tested during a 1-year cost-free verification phase.

FreeVaP is a downsized version of VaP 4.0 offered for free to everyone interested in probabilistic analysis.

The software may be requested via www.ponti.eu/vap. A download link will be sent back where all necessary parts and instructions for installation can be streamed down supplemented with a number of examples.

Developer and Distributor: PSP GmbH, Feldkirchen, Austria
For information: http://www.petschacher.at

7.53 SAPOS

In 2015, Dr. H. Karadeniz wrote the following:

SAPOS stands for Stochastic Analysis Program for Offshore Structures. It is a general-purpose 3D beam-element program for static and dynamic analyses. It calculates both deterministic and stochastic responses of any type framed structure. Stochastic response characteristics are calculated by carrying out an efficient spectral analysis procedure.

Since it is a program for framed structural analysis, offshore steel structures can be analyzed mainly by SAPOS under random sea waves or earthquake loads. SAPOS is capable to take into account joint flexibilities (partly member-connections), water-structure and soil-structure interactions phenomena. For offshore steel structures, it calculates fatigue damages and fatigue lifetimes for both narrow and non-narrow banded stress processes. For the fatigue reliability analysis, an extensive uncertainty modeling is available in SAPOS and the reliability estimate is carried out by FORM. Due to efficient uncertainty modeling SAPOS can perform the reliability analysis of large offshore structures in seconds.

SAPOS is available in a PC-version and has user-friendly input/output environments. Its graphical facility enables users to visualize input information and response outputs.

Developer and Distributor: H. Karadeniz, formerly Civil Engineering Department of Delft University of Technology, TUD.

For information: e-mail: h.karadeniz.tr@gmail.com

7.54 PROB2B

In 2016 Dr. Wim Courage wrote the following:

Prob2B is a probabilistic toolbox, capable of being coupled to other software for problem oriented calculations in the limit state function evaluations. The toolbox contains a number of structural reliability methods for examining limit states composed of model outcomes: general methods such as plain numerical integration, Crude Monte Carlo, Increased Variance Sampling, Directional Sampling and FORM/SORM are implemented. As a more dedicated method, DARS is available which is a combination of Directional Sampling with a Response Surface that is created and updated during sampling. The response surface enables efficiency in calculation time as, under appropriate conditions, intermediate samples can be taken from this surface instead of from expensive model calculations.

Probabilistic Distribution functions describing random variables can be chosen from a library with widely used types (for example normal, Lognormal, Beta, Gumbel) or be user defined by means of tabulated probability data.

The current version of Prob2B has a graphical user interface for selecting models and defining limit states. Interfaces to Excel and Matlab are enabled as well as some dedicated stand alone executables. New developments are being done in Python scripting in such a way that users will be able to make their own interfacing of the Prob2B structural reliability methods with their own (stand-alone) external models like, e.g., finite element packages.

Developer and Distributor: TNO, Department of Structural reliability, Delft.

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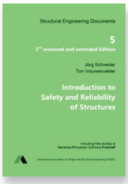

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Introduction to Safety and Reliability of Structures

Society expects that buildings and other structures are safe for the people who use them or who are near them. The failure of a building or structure is expected to be an extremely rare event. Thus, society implicitly relies on the expertise of the professionals involved in the planning, design, construction, operation and maintenance of the structures it uses.

Structural engineers devote all their effort to meeting society's expectations efficiently. Engineers and scientists work together to develop solutions to structural problems. Given that nothing is absolutely and eternally safe, the goal is to attain an acceptably small probability of failure for a structure, a facility, or a situation. Reliability analysis is part of the science and practice of engineering today, not only with respect to the safety of structures, but also for questions of serviceability and other requirements of technical systems that might be impacted by some probability.

The present volume takes a rather broad approach to safety and reliability in Structural Engineering. It treats the underlying concepts of safety, reliability and risk and introduces the reader in a first chapter to the main concepts and strategies for dealing with hazards. The next chapter is devoted to the processing of data into information that is relevant for applying reliability theory. Two following chapters deal with the modelling of structures and with methods of reliability analysis. Another chapter focuses on problems related to establishing target reliabilities, assessing existing structures, and on effective strategies against human error. The last chapter presents an outlook to more advanced applications. The Appendix supports the application of the methods proposed and refers readers to a number of related computer programs.

This book is aimed at both students and practicing engineers. It presents the concepts and procedures of reliability analysis in a straightforward, understandable way, making use of simple examples, rather than extended theoretical discussion. It is hoped that this approach serves to advance the application of safety and reliability analysis in engineering practice.

The book is amended with a free access to an educational version of a Variables Processor computer program. FreeVaP can be downloaded free of charge and supports the understanding of the subjects treated in this book.

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